Unified Explanation of Envelope Broadening and Maximum-Amplitude Decay of High-Frequency Seismograms based on the Envelope Simulation using the Markov Approximation: Forearc Side of the Volcanic Front in Northeastern Honshu, Japan

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Abstract  During propagation through heterogeneous lithosphere, S-waves impulsively radiated from a source increase in duration and decrease in maximum amplitude with increasing travel distance. We term these phenomena “envelope broadening” and “maximum-amplitude decay”, respectively. The present study is the first attempt to make a unified explanation of the envelope broadening and the maximum-amplitude decay based on wave scattering in random media. S-wave envelopes observed at the forearc side of the volcanic front in northeastern Honshu, Japan have following characteristics in the frequency range from 2 to 16Hz in the distance range from 75 to 410 km; envelope duration increases with increasing travel distance in proportion to the distance to the power 1.7 to 1.8 and maximum amplitude decreases with increasing travel distance in proportion to the distance to the power –3 to –2. These observations are modeled by considering small-angle wave-scattering around the forward direction in the media having randomly fluctuating wave-velocity with frequency-dependent attenuation. The joint analysis of envelope duration and maximum-amplitude is superior in estimating medium inhomogeneity and attenuation over the analysis of envelope duration or maximum-amplitude alone. Assuming the von Kármán-type random media, we estimate the power spectral density function of S-wave velocity fluctuation as $P(m) \approx 0.004m^{-5.0} - 0.01m^{-4.0}\text{km}^{-3}$ in the range between $0.5 < m < 50\text{ km}^{-1}$, where $m$ is the wavenumber of the inhomogeneity. The theoretical envelope calculated from the estimated random inhomogeneity and attenuation successfully gives unified explanation of the envelope broadening and the maximum-amplitude decay in northeastern Honshu, Japan.

Key words: envelope broadening, maximum-amplitude decay, lithosphere, heterogeneity, random media, parabolic approximation
1. Introduction

In a regional seismic network, high-frequency seismograms (> 1Hz) of a local small earthquake, especially around the maximum amplitude, are different from station to station. Due to this complexity, it is difficult to evaluate the path effect by deterministic approaches. Hence, regression analyses and stochastic methods are often used for the evaluation of the seismic wave propagation. Duration and maximum amplitude of seismograms have been considered as important parameters for the quantification of seismograms and have been investigated statistically in a number of papers.

Duration of seismogram has been investigated for the purpose of the prediction of strong ground motion or the estimation of the inhomogeneity of the lithosphere. In the simulation of ground motion referred to as stochastic method, the duration of seismogram envelope is one of the important parameters [e.g. Hanks and McGuire, 1981; Boore, 1983], where an empirical function proposed by Saragoni and Hart [1974] is usually used to represent the envelope shape. The duration of envelope, generally depending on both source and path, increases with increasing travel distance [e.g. Atkinson and Boore, 1995]. In other words, the envelope broadens on the time axis with increasing travel distance. We call this \textit{envelope broadening}. Sato [1989] quantitatively explains the envelope broadening by considering small-angle scattering of waves around the forward direction caused by the random inhomogeneity of wave velocity. The theoretical envelope is simulated by using the Markov approximation which is a stochastic approximation applied to the parabolic wave-equation [e.g. Ishimaru, 1978]. Fehler et al. [2000] confirms the validity of the parabolic approximation and the Markov approximation by comparing the envelopes calculated using those approximations and those from waveforms of the 2-D finite difference simulation. Sato [1989] suggests that the duration of seismogram is a good measure for representing the inhomogeneity of the lithosphere. Following this work, some methods for
estimating random inhomogeneity have been developed [e.g. Scherbaum and Sato, 1991; Obara and Sato, 1995]. Another envelope simulation method based on the radiative transfer theory has also been developed and applied to observations for the simulation of envelope broadening [e.g. Gusev and Abubakirov, 1996; Gusev and Abubakirov, 1999a, b; Petukhin and Gusev, 2003].

Maximum amplitude of seismogram has also been considered as one of the most important parameters for the quantification of seismograms. As pioneering works, maximum-amplitude decay with travel distance is investigated by Wadati [1931] and Richter [1935] for the purpose of the estimation of event magnitudes. Since then, many papers have investigated the maximum-amplitude decay not only for the determination of magnitudes but also for the strong ground-motion prediction. A functional form $\log A = B_o + C_o \log r + D_o r$ is generally employed in regression analyses to acquire the relation between maximum amplitude $A$ and travel distance $r$, where $C_o$ is related to the geometrical spreading and $D_o$ is related to quality factor $Q$ [e.g. Joyner and Boore, 1981; Fukushima and Tanaka, 1990; Atkinson and Boore, 1995]. Recently, travel-distance dependent geometrical spreading is introduced for the more precise prediction of maximum-amplitude [e.g. Atkionson and Mereu, 1992; Raoof et al., 1999; Malagnini et al., 2000; Malagnini et al., 2002]. On the other hand, a simple power-law function $\log A = B_o + C_o \log r$ is known as a good approximation in spite of a small number of regression coefficients [Noguchi, 1990]. In this equation, a parameter $C_o$ gives a feature of maximum-amplitude decay. The Japan Meteorological Agency has used the power-law function with $C_o = -1.73$ for the magnitude determination of small events around Japan [Tsuboi, 1954; Watanabe, 1971]. Fukushima and Tanaka [1990] also conclude that $C_o = -1.7$ from the analysis of 22,000 horizontal seismograms observed by modern seismic networks all over Japan. A careful analysis of the spectral amplitude decay around Kanto, Japan indicates that the amplitude decreases more
rapidly with travel distance in high frequencies than that in low frequencies: $C_0 = -1.5$ at 1Hz and $C_0 = -2.0$ at 10Hz [Kinoshita, 1986].

As explained above, the envelope duration and the maximum amplitude have been used as good measures for the quantification of high-frequency regional seismograms. However, the envelope broadening and the maximum-amplitude decay have been studied independently or empirically. The unified explanation of those phenomena based on wave scattering process has never been done. It is because the conventional envelope simulation method for plane waves was not realistic enough [e.g. Sato, 1989].

The objective of this article is the unified explanation of the envelope duration and the maximum-amplitude decay of high-frequency S-wave envelopes. We use the theoretical envelope of Saito et al. [2002a] which is formulated under the assumption of spherical wave propagation from a point source in 3-D random media [Shishov, 1974]. At first, we investigate the travel-distance and frequency dependence of envelope duration and maximum amplitude from S-wave envelopes observed in the forearc side of the volcanic front in northeastern Honshu, Japan. In this region, high-quality data set of a dense seismic network is now available, whereas Saito et al. [2002a] analyzed seismograms observed at a single station in this region. Then, we simulate the observed duration and maximum amplitude simultaneously based on the theoretical envelope by using an appropriate statistical property of the velocity inhomogeneity and the attenuation of the lithosphere.
2. High-Frequency Envelopes Observed at the Forearc Side of the Volcanic Front in Northeastern Honshu, Japan

2.1 Tectonic Setting in Northeastern Honshu

Northeastern Honshu forms an island arc associated with the westerly subducted Pacific plate. As shown in Figure 1, quaternary active volcanoes (solid triangles) are distributed in parallel to the trench axis, and their east end is clearly bounded by the volcanic front (dashed curve), which divides the island into two parts; the east side is forearc side, and the west side backarc side. It is a well known fact that the amplitude of the direct S-wave rapidly decreases when the wave propagates into the backarc side [e.g. Umino and Hasegawa, 1984]. Another difference was also recognized in the envelope shape; S-wave envelopes observed in the forearc side show no significant frequency dependence, while those in the backarc side have a longer duration with increasing frequency, which indicates the difference in statistical property of the medium inhomogeneity [Obara and Sato, 1995; Saito et al, 2002b]. Recent seismological studies have revealed three-dimensional features of wave velocities and attenuation structures by tomographic methods [e.g. Umino and Hasegawa, 1984; Petukhin and Irikura, 2000; Tsumura et al., 2000; Nakajima et al., 2001]. Those results indicate that the wedge mantle of the forearc side is characterized by high velocity and weak attenuation, while that of the backarc side by low velocity and strong attenuation. As shown above, many observational facts indicate the difference in seismic structure between the forearc side and the backarc side in this region.

2.2 Data

We restrict a target region to the forearc side in this study, because we will assume that the random inhomogeneity of the medium is uniform in space. In Figure 1, stations and
events used in this study are plotted by diamond and star symbols, respectively. We use seismograms recorded at 57 stations of High-Sensitivity Seismograph Network Japan (Hi-net), which are deployed by the National Research Institute for Earth Science and Disaster Prevention (NIED). At each station, a three-component velocity-type seismometer of natural frequency 1Hz is installed at the bottom of a borehole with depth more than 100m. High quality records free from reverberations due to soft-soil deposits are obtained by the Hi-net. The signals are digitized with a 100Hz sampling frequency with a 27 bit-resolution. Eight events for analysis are selected from the following point of view. Intermediate-depth events are appropriate to minimize the effect of surface waves and refracted waves from Moho and Conrad discontinuities. Events of magnitude 3.5 – 4.5 are appropriate since they have large enough signal-to-noise ratio and short enough source durations. Table 1 lists the eight events used in the analysis.

2.3 Measurement of Envelope Duration and Maximum Amplitude

To quantify observed seismograms, we introduce two parameters: envelope duration $t_q$ and maximum amplitude $A_{\text{max}}$ of Root-Mean-Square envelope. We investigate their travel-distance and frequency dependence.

The Root-Mean-Square envelopes are made from two horizontal-component seismograms as follows. After the correction of the instrumental response, bandpass-filtered traces with center frequencies 2, 4, 8 and 16Hz are made. For each frequency band, squared bandpass-filtered traces in NS and EW components are summed up. The trace is smoothed by applying the low-pass filter of which the corner frequency is the same as the center frequency of the bandpass-filter. The resultant trace is referred to as Mean-Square (MS) envelope, and the square root of the MS envelope as Root-Mean-Square (RMS) envelope,
hereafter. We analyze the envelopes after the S-wave onset which is referred to as S-wave envelopes.

The envelope duration \( t_q \) is defined as the lag time between the S-wave onset and the time when RMS envelope decays to the half of the maximum amplitude. Figure 2 shows an example of velocity seismogram and RMS envelope at 8Hz band for the #5 event of Table 1. A fine vertical dashed-line indicates the S-wave onset, and a coarse vertical dashed-line the time when the RMS envelope decays to the half of the maximum amplitude. We measure \( t_q \) and \( A_{\text{max}} \) of RMS envelope at each frequency band. The total number of the set of \( t_q \) and \( A_{\text{max}} \) we measured is 388 for each frequency band. The hypocentral distance ranges from 75 to 410km. In this study, the effect of source radiation-pattern on \( t_q \) and \( A_{\text{max}} \) is not considered since the effect is small in the high-frequency band of present concern [e.g., Liu and Helmberger, 1985; Satoh, 2002].

### 2.4 Observed Envelope Broadening and Maximum-Amplitude Decay

Figure 3 shows the log-log plots of the envelope duration \( t_q \) [s] (dot) against the hypocentral distance \( r \) [km] for each frequency band. The data for all event-station pairs are plotted together in each figure. Although data points scatter widely, the envelope duration on average increases with increasing propagation distance. Dashed lines are the regression lines given by

\[
\log t_q = B_q + C_q \log r .
\]  

(1)

Estimated regression coefficients and the standard deviation from the regression line are shown at the bottom of each figure. The value of \( C_q \) ranges from 1.72 to 1.77, and the
The value of $q_B$ ranges from –3.22 to –3.05. There is no significant frequency dependence in the regression coefficients, while the standard deviation shows a decrease with increasing frequency. The $C_q$ values estimated in this study are almost the same as the value 1.74 – 1.82 obtained from one station at hard-rock site in the forearc side of northeastern Honshu [Saito et al. 2002a].

Figure 4 shows the log-log plots of the maximum amplitude $A_{max}$ (dot) against hypocentral distance $r$ [km] at 8Hz band for the eight events. Solid lines are the regression lines given by

$$\log A_{max} = B_M + C_m \log r$$ (2)

for each event. The values of $B_M$, which depend on the magnitude, are arbitrarily adjusted in the figure. The values of $C_m$ are listed in Table 2 for each event for each frequency band. The $C_m$ value averaged over the eight events shows a decrease from –2.2 to –3.2 with increasing frequency; the maximum amplitude $A_{max}$ decreases more rapidly as frequency increases. The $C_m$ values estimated in this analysis are smaller than the value –1.5 to –2.0 estimated by other researchers in Japan [e.g. Tsuboi, 1954; Watanabe, 1971; Kinoshita, 1986; Fukushima and Tanaka, 1990]. This discrepancy may be attributed to some reasons: the difference between event depths, analyzed areas, frequency bands, and so on. The most contributory factor would be the difference between the travel-distance ranges of data sets. The maximum amplitude, in general, decreases more rapidly in long travel distance than in short travel distance [see Figure 11, Fukushima and Tanaka, 1990]. Therefore, the $C_m$ value estimated from the events located in wide travel-distance range will be larger than that from the events located only in long travel-distance. This study analyzes the events located
only in long travel-distance compared to the previous studies, so that, the $C_{\nu}$ values in this study are smaller than those in previous studies.

3. Simulation Method for Ensemble-Average Wave Envelope in Random Media

The envelope around the maximum amplitude is composed of a number of scattered waves [Figure 2]. Therefore, by considering wave scattering due to 3-D medium inhomogeneity, we will simulate envelope-broadening and maximum-amplitude decay to explain the observed ones. This section briefly explains the simulation method for ensemble-average wave-envelope in random media [Saito et al., 2002a].

We assume that wave is isotropically radiated from a point source and propagates through a 3-D inhomogeneous medium. The inhomogeneity causes scattering and diffraction of the wave. As a result, a wave, which is impulsive at the source, increases in duration and decreases in maximum amplitude as propagation distance increases. When the wavelength is shorter than the characteristic scale of the inhomogeneity, conversion scattering between P and S waves can be neglected. Hence, it is acceptable to describe the principal characteristics of seismic wave propagation by using the wave equation for wavefield $u(x,t),$

$$\left(\Delta - \frac{1}{V(x)}\frac{\partial^2}{\partial t^2}\right)u(x,t) = 0.$$  (3)

The wave velocity $V(x)$ is written as $V(x) = V_0[1 + \xi(x)],$ where $V_0$ is the background velocity, and $\xi(x)$ is fractional fluctuation of wave velocity.

An ensemble of random media $\{\xi(x)\}$ such that $\langle\xi(x)\rangle = 0,$ is considered, where the angular bracket means the ensemble average. Random media are statistically characterized
by the autocorrelation function (ACF) of $\xi(x)$ or its Fourier transform, the power spectral density function (PSDF). We assume that $\xi(x)$ is a homogeneous and isotropic random function of location $x$. The von Kármán-type random media are used to represent the random inhomogeneity. Their PSDFs are given by

$$P(m) = \frac{\Gamma\left(\frac{3}{2}\right)}{\Gamma\left(\frac{1}{2}\right)(1 + m^2)^{3/2}} + \kappa \Gamma\left(\kappa + \frac{3}{2}\right)$$

where $\Gamma$ is the gamma function and $m = |m|$ is wavenumber. The PSDF is determined by three parameters. MS value of fractional velocity fluctuation $\varepsilon = \langle \xi^2 \rangle$ is a measure of the magnitude of the inhomogeneity, correlation distance $a$ is a measure of the characteristic scale of the spatial variation of $\xi$, and the order $\kappa$ controls the power index at large wavenumbers $m \gg a - 1$ where the PSDF asymptotically obeys a power-law. Figure 5 (a) plots the PSDF in logarithmic scale.

When the small-angle (forward) scattering of waves dominates over the large-angle scattering, the wave equation is approximated by the parabolic equation [e.g. p.230 Sato and Fehler, 1998]. We use local Cartesian coordinates at a large distance $r$ from the source, where one axis is chosen to be in the radial direction and the other two axes $r_\perp$ are in the transverse plane which is tangent to the sphere of radius $r$ [Figure 6]. Waves are written as a superposition of harmonic spherical waves of amplitude $U$ at a angular frequency $\omega$ as

$$u(r_\perp,r) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{U(r_\perp,r,\omega)}{r} e^{i(r-\omega)d\omega}, \quad (5)$$
where wavenumber $k = \alpha / \nu_0$. In order to derive wave envelope in random media we define the two-frequency mutual coherence function (TFMCF) on the transverse plane at distance $r$ [e.g. Ishimaru, 1978],

$$\Gamma_2(r_{\perp1}, r_{\perp2}, r, \omega_1, \omega_2) = \langle U(r_{\perp1}, r, \omega_1) U^*(r_{\perp2}, r, \omega_2) \rangle$$

(6)

where the asterisk means complex conjugate. Since the random media are statistically homogeneous, $\Gamma_2$ is independent of the center of transverse location $r_c = (r_{\perp1} + r_{\perp2})/2$ and depends on the difference of transverse location $r_{\perp d} = r_{\perp1} - r_{\perp2}$. The center-of-mass and difference coordinates in the wavenumber are $k_c = (k_{1} + k_{2})/2$ and $k_d = k_{1} - k_{2}$, respectively ($k_d << k_c$). Corresponding angular frequency is also used. The master equation of $\Gamma_2$ is derived from the parabolic equation [(8) in Saito et al. 2002a]. In this equation, the contribution of the inhomogeneity is given by

$$A(r_{\perp d}) = \int_{-\infty}^{\infty} \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(m) \exp[i m (r_{\perp d} + z e_z)] \, dm .$$

(7)

We may factor $\Gamma_2$ into the product $\Gamma_2 = \phi \exp(-k_{d}^2 A(0)/2)$. The term $\exp(-k_{d}^2 A(0)/2)$ means the wandering effect which does not correspond to the broadening of single-realized seismograms [e.g. Lee and Jokipii, 1975; p. 247, Sato and Fehler, 1998]. We use $\phi \Gamma_2$ instead of $\Gamma_2$ in the following. For quasi-monochromatic waves, the master equation for $\phi \Gamma_2$ is written as,

$$\frac{\partial}{\partial r} \phi \Gamma_2(r, \theta_d, \omega_1, \omega_2) + i \frac{k_d}{2k_c^2} \frac{1}{r^2} \left( \frac{\partial^2}{\partial \theta_d^2} + \frac{1}{\theta_d} \frac{\partial}{\partial \theta_d} \right) \phi \Gamma_2 + k_d^2 [A(0) - A(r, \theta_d)]_0 \phi \Gamma_2 = 0 ,$$

(8)
where the difference angle is \( \theta_d = |\mathbf{r}_d|/r \). The MS envelope in a narrow-frequency band \( \Delta \omega_c \) around the center angular-frequency \( \omega_c \) is written as

\[
I(r, t; \omega_c) = \left( |u(\mathbf{r}_d, r, t)| \right)^2 = \frac{\Delta \omega_c}{(2\pi)^2 r^2} \int_0^\infty \Gamma_2(\theta_d = 0, r, \omega_c, \omega_c) e^{-i\omega_c(\mathbf{r}_d - \mathbf{r})} d\omega_c.
\]

For an isotropic source radiation with total source wave-energy \( W \) within this angular frequency band,

\[
I(r \to 0, t; \omega_c) = \frac{W}{4\pi V_0 r^2} \delta \left( t - \frac{r}{V_0} \right),
\]

we choose the initial condition of \( \Gamma_2 \) as

\[
\Gamma_2(\theta_d = 0, r, \omega_c, \omega_c) = \frac{W}{2\Delta \omega_c V_0}.
\]

By solving the master equation of the TFMCF (8) under the initial condition of (11) and inserting the TFMCF into (9), we can synthesize theoretical MS envelope.

The above simulation method assumes small-angle scattering around the forward direction. In the case that wavelength is much smaller than the characteristic scale of the inhomogeneity \( ak \gg 1 \) (\( k \) is wavenumber) and the inhomogeneity spectrum is poor in short wavelength components, the small-angle scattering dominates over the large-angle scattering [e.g. p. 92, Sato and Fehler, 1998]. In our data set, the lower limit of wavenumber is \( k = 3 \text{ km}^{-1} \) (\( f_c = 2 \text{ Hz} \), \( V_0 = 4 \text{ km/s} \)). Some papers reported that the characteristic scale of the inhomogeneous lithosphere is larger than a few km [see the review Wu and Aki, 1988].
Hence, the condition \( ak >> 1 \) would be satisfied in our data set. In addition, numerical tests indicate that the theoretical envelope is reliable for synthesizing the early part of envelopes even when the method fails to simulate the later part of envelope due to the short wavelength spectral components of the von Kármán-type random media [Saito et al, 2003]. Therefore, we may use the theoretical envelope for the analysis of the envelope broadening and the maximum-amplitude decay.

In Figure 7, theoretical MS envelopes for a unit source radiation synthesized by our simulation method are plotted. The MS amplitude is normalized by the geometrical spreading factor and a characteristic time \( t_M \), and the reduced time is normalized by \( t_M \). The characteristic time \( t_M \), for frequency \( f_c \) and at travel distance \( r \), is defined as

\[
    t_M = F_0(\epsilon, a, \kappa)(2\pi f_c)\left[\frac{\kappa}{\rho(\kappa)}\right]^{\frac{1}{4}} \frac{2}{r^{\frac{1}{\rho(\kappa)}}},
\]

where

\[
    F_0(\epsilon, a, \kappa) = \frac{1}{2} C(\kappa)^2 \frac{2}{\rho(\kappa)} F_0^{1-\frac{4}{\rho(\kappa)}} \left(\frac{\epsilon^{\frac{\rho(\kappa)-1}{\rho(\kappa)}}}{2^{\frac{1}{\rho(\kappa)}}} \right)^\frac{2}{\rho(\kappa)}.
\]

The dimension of \( t_M \) and \( F_0 \) are \([s]\) and \([s^{-1/4}/km^{-1/2/\rho}]\) respectively, when we use the dimension \([s]\) in time and \([km]\) in length. Parameters \( C(\kappa) \) and \( \rho(\kappa) \) depend on the order \( \kappa \) of von Kármán type random media as listed in Table 3 [after Saito et al. 2002a]. In Figure 8, as an example, theoretical RMS envelopes at hypocentral distances 100, 150, 200, 250 and 300km are plotted for the case of \( \kappa = 0.8 \), \( \epsilon = 0.05 \) and \( a = 5 \) km. A broken curve shows the maximum amplitude decay due to the geometrical spreading factor which is
proportional to the reciprocal of travel distance. As hypocentral distance increases, theoretical envelope increases in duration and decreases in maximum amplitude. The maximum amplitude decreases more rapidly than the broken curve due to scattering of waves.

The theoretical envelopes shown above include the contribution of small-angle scattering or diffraction of waves, although they do not contain the effect of intrinsic absorption and large-angle scattering. Both the intrinsic absorption and the large-angle scattering works as energy loss in the early part of the envelopes. To include phenomenologically those effects, we multiply \( e^{-bx} \) to \( \left| u \right|^2 \), where \( b \) is referred to as attenuation coefficient hereafter.

4. Dependence of the Envelopes on the Medium Inhomogeneity and Attenuation

Based on the theoretical envelope presented above, we will simulate the envelope broadening and maximum-amplitude decay observed at northeastern Honshu estimating the appropriate random inhomogeneity and attenuation. This simulation needs six model parameters; \( F_0 \) and \( \kappa \) for the medium inhomogeneity, and \( b_1, b_2, b_3, \) and \( b_4 \) for the attenuation at 2, 4, 8 and 16 Hz bands. The number of the parameters is too large to be estimated by a simple grid search. For the development of a new estimation method, it is helpful to get an insight into the dependence of the envelope on the medium inhomogeneity and attenuation.

In this section, restricting the data set at 16Hz band, we investigate the dependence of the envelope on the medium parameters \( F_0 \) and \( b_4 \). As examples, fixing parameter \( \kappa \) to be 0.8, we show the analyses of the envelope broadening, of the maximum-amplitude decay,
and of the joint analysis of the envelope broadening and the maximum-amplitude decay. The value $\kappa = 0.8$ will be justified in the following section.

**Envelope Broadening**

The relation between the envelope broadening and medium parameter $F_0$ and $b_4$ is investigated by evaluating the sum of squared residuals between the observation and the theoretical calculation of $t_q$:

$$ R_{t_q} = \sum_{n=1}^{N_{\text{total}}} \left[ \log t_{q,\text{obs}} - \log t_{q,\text{cal}}(F_0, b_4) \right]^2 ,
$$

(14)

where $t_{q,\text{obs}}$ is observed $t_q$ value at the $n$-th data and $t_{q,\text{cal}}$ is measured from the theoretical envelope calculated using the model parameters $F_0$ and $b_4$. The total number of the data $N_{\text{total}}$ is 388. The search range of $F_0$ is from $1.5 \times 10^{-4} \ [s^{1.02} \text{km}^{-2.01}]$ to $3.0 \times 10^{-4} \ [s^{1.02} \text{km}^{-2.01}]$ with a step of $0.05 \times 10^{-4} \ [s^{1.02} \text{km}^{-2.01}]$, and the range of $b_4$ is from 0.00 [$s^{-1}$] to 0.16 [$s^{-1}$] with a step of 0.01 [$s^{-1}$].

Figure 9 (a) shows the contour map of the residual value (14) in terms of $F_0$ and $b_4$. The pattern of the contour map indicates a trade-off between $F_0$ and $b_4$; the case of large $F_0$ and large $b_4$ is equivalent to the case of small $F_0$ and small $b_4$. The value (14) is minimized at $F_0 = 2.3 \times 10^{-4} \ [s^{1.02} \text{km}^{-2.01}]$ and $b_4 = 0.07 \ [s^{-1}]$ (star), which are considered as the best model parameters to simulate the envelope broadening observed at 16Hz band. The standard errors of the model parameters are estimated as $\Delta F_0 = 0.14 \times 10^{-4} \ s^{1.02} \text{km}^{-2.01}$ and $\Delta b_4 = 0.017 \text{s}^{-1}$ by using the bootstrap method [Efron and Tibshirani, 1986], where we used
100 synthesized data sets of size $N_{\text{total}}$, each of which is randomly sampled from the actual data set \( \left( \log t_{q,1}^{\text{obs}}, \ldots, \log t_{q,N_{\text{total}}}^{\text{obs}} \right) \).

**Maximum-Amplitude Decay**

The relation between the maximum-amplitude decay and medium parameter $F_0$ and $b_4$ is investigated by evaluating the sum of squared residuals between the observation and the calculation of $A_{\text{max}}$:

\[
R_{\text{d,4}} = \sum_{i=1}^{N_e} \sum_{j=1}^{N_s} \left( \log A_{\text{max},ij}^{\text{obs}} - \log A_{\text{max},ij}^{\text{cal}}(F_0, b_4; W_i) \right)^2,
\]

where $A_{\text{max},ij}^{\text{obs}}$ is the maximum amplitude observed at the $j$th station for the $i$th event, $A_{\text{max},ij}^{\text{cal}}$ is measured from the theoretical envelope calculated using model parameters $F_0$ and $b_4$. The number of events and the number of stations for the $i$-th event are represented by $N_e$ and $N_{s,i}$, respectively. The relation between $N_e$, $N_{s,i}$ and $N_{\text{total}}$ is given by

\[
N_{\text{total}} = \sum_{i=1}^{N_e} N_{s,i} = 388.
\]

Source radiated energy $W_i$ at 16Hz band for the $i$-th event is estimated from the maximum-amplitude decay curve. The search ranges of $F_0$ and $b_4$ are the same as those used in the envelope broadening analysis.

Figure 9 (b) shows the contour map of the residual value (15) in terms of $F_0$ and $b_4$. The pattern of the contour map indicates that the range of $b_4$ is well constrained but the range of $F_0$ is not constrained at all. The value (15) is minimized at the model parameters $F_0 = 3.0 \times 10^{-4} \text{ s}^{1.02} \text{ km}^{-2.01}$ and $b_4 = 0.04 \text{ s}^{-1}$ (star), which are considered as the best model parameters to simulate the maximum amplitude decay observed at 16Hz band. The standard error of $b_4$ is estimated as $\Delta b_4 = 0.005 \text{ s}^{-1}$ by using the bootstrap method, where we used 100
synthesized data sets of size \(N_{\text{total}}\), each of which is randomly sampled from the actual data set \(\{\log A_{\text{max},1\text{,obs}}, \ldots, \log A_{\text{max},N_{\text{obs}},i\text{,obs}}\}\).

**Joint Analysis of Envelope Broadening and Maximum-Amplitude Decay**

The results of the independent analyses for the envelope broadening and the maximum-amplitude decay suggest that both \(F_0\) and \(b_4\) may be constrained in a reasonably small region by a joint analysis of \(t_q\) and \(A_{\text{max}}\). To take account of the envelope broadening and the maximum-amplitude decay simultaneously, we introduce the following residual value as a combination,

\[
R_i = \frac{1}{\sigma_i^2} R_{q,i} + \frac{1}{\sigma_A^2} R_{A,i},
\]

where \(\sigma_i^2\) and \(\sigma_A^2\) are the variances of \(\log t_{q,i\text{,obs}}\) and \(\log A_{\text{max},i\text{,obs}}\), respectively. In this study, those variances are defined as,

\[
\sigma_i^2 = \frac{R_{q,i\text{,min}}}{N_{\text{total}}}, \quad \sigma_A^2 = \frac{R_{A,i\text{,min}}}{\sum_{i=1}^{N_{\text{obs}}} N_{t,q,i}}
\]

where \(R_{q,i\text{,min}}\) is the minimum value of \(R_{q,i}\) [(14)] as a function of \(F_0\) and \(b_4\), and \(R_{A,i\text{,min}}\) is the minimum value of \(R_{A,i}\) [(15)]. Using (17), we obtain the value of (16) as

\[
R_i = N_{\text{Total}} \cdot R_{i\text{,min}},
\]

where

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ENVELOPE BROADENING AND AMPLITUDE DECAY

\[ R_{\text{min}} = \frac{R_{q_{\text{min}}}}{R_{q_{\text{max}}}} + \frac{R_{q_{\text{max}}}}{R_{q_{\text{min}}}} = 1 \sum_{n=1}^{N} \sum_{i=1}^{N} \log t_{q_{i}}^{\text{obs}} - \log t_{q_{i}}^{\text{cal}}(F_{0}, b_{4})^2 \]

The residual value (19) is evaluated as a function of \( F_{0} \) and \( b_{4} \). The search ranges of \( F_{0} \) and \( b_{4} \) are the same as those used in the previous analyses.

Figure 9 (c) shows the contour map of the residual value (19) in terms of \( F_{0} \) and \( b_{4} \). The minimum value of (19) is taken at \( F_{0} = 2.15 \times 10^{-4} \text{ s}^{-1} \text{ km}^{-2.01} \) and \( b_{4} = 0.04 \text{ s}^{-1} \) (star), which are considered as the best model parameters to simulate both the envelope broadening and the maximum-amplitude decay observed at the 16Hz band. The standard errors of the best estimates are estimated as \( \Delta F_{0} = 0.08 \times 10^{-4} \text{ s}^{-1} \text{ km}^{-2.01} \) and \( \Delta b_{4} = 0.005 \text{ s}^{-1} \) by using the bootstrap method. For the bootstrap analysis, we used 100 synthesized data sets of size \( 2N_{\text{total}} \), each of which is randomly sampled from \( (\log t_{q_{i}}^{\text{obs}}, \log t_{q_{i}}^{\text{obs}}) \) and \( (\log A_{\text{max},q_{i}}^{\text{obs}}, \log A_{\text{max},q_{i}}^{\text{obs}}) \). Figure 9 (c) clearly shows that the joint analysis solves the trade off between \( F_{0} \) and \( b_{4} \) appearing in the analysis of \( t_{q} \) [see Figure 8(a)]. The joint analysis estimates the model parameters with better accuracy.

From the above analyses, we get to know that the maximum-amplitude decay is strongly controlled by attenuation coefficient \( b \), but is almost independent of parameter \( F_{0} \). Furthermore, the joint analysis of \( t_{q} \) and \( A_{\text{max}} \) is more useful than the individual analyses. These features are also recognized at other frequency bands.
5. Estimation of Velocity Inhomogeneity and Attenuation Structure Beneath N. E. Honshu

In this section, using the data set of $t_q$ and $A_{max}$ for all the frequency bands 2, 4, 8 and 16 Hz, we estimate the PSDF of random inhomogeneity of S-wave velocity and the attenuation beneath northeastern Honshu. The model parameters are $F_0$, $\kappa$ and attenuation coefficient $b_k$ at $f_c = 2^k$ Hz ($k = 1, 2, 3, 4$). Source radiated energy $W_d$ for the $k$-th band for $i$-th event is also needed for the calculation of the theoretical envelopes. Those model parameters are too many to be estimated by a joint analysis such as shown in the previous section, so that a two-step grid-search is employed for the estimation. The method is developed based on the characteristic dependence of the envelope on the medium parameters. We estimate the six parameters $F_0$, $\kappa$ and $b_k$ ($k = 1, 2, 3, 4$) by the two-step grid-search as follows.

We consider the case for a value of $\kappa$. In the 1st-step grid search, we search appropriate values $b_k$ from the maximum-amplitude decay so as to minimize the sum of the squared residuals:

$$R_{Ak} = \sum_{i}^N \sum_{j}^N \left[ \log A_{max,ijk}^{obs} - \log A_{max,ijk}^{cal}(F_0, b_k; W_d) \right]^2,$$

(20)

where $A_{max,ijk}^{obs}$ is the maximum amplitude of RMS envelope in the $2^k$ Hz band observed at the $j$-th station for the $i$-th event, and $A_{max,ijk}^{cal}$ is the maximum amplitude of theoretical RMS envelope for model parameters $F_0$, $b_k$, and $W_d$. Source radiated energy $W_d$ for the $k$-th band for the $i$-th event is estimated by using the maximum amplitude decay curve calculated from the model parameters $F_0$ and $b_k$. The search range of $b_k$ is from 0 [s$^{-1}$] to
0.16 [s⁻¹] with a step of 0.01 [s⁻¹]. From the analysis in the previous section, we know that the slope of theoretical maximum-amplitude against hypocentral distance strongly depends on parameter $b_k$, while the average level of $A_{\text{max},n,k}^{col}$ strongly depends on parameters $F_0$ and $W_{ik}$. We can estimate parameter $b_k$ independently from the parameters $F_0$ and $W_{ik}$ evaluating the value of $R_{\alpha,k}$ for each frequency band.

In the second-step grid search, we estimate the value of $F_0$ from the envelope broadening so as to minimize the sum of the squared residuals:

$$R_q = \sum_{k=1}^{4} \sum_{n=1}^{N} \left[ \log t_{q,n,k}^{\text{obs}} - \log t_{q,n,k}^{\text{cal}}(F_0,b_{n,k}) \right]^2,$$

where $t_{q,n,k}^{\text{obs}}$ represents the envelope duration for the $n$-th RMS envelope in the $2^q$ Hz band. For the attenuation coefficients $b_{n,k}$ at $2^q$ Hz ($k=1, 2, 3, 4$) bands, we use the values estimated in the first-step grid search. The search range of $F_0$ is from $0.1 \times 10^{-4}$ $[s^{-1} \text{km}^{-1/2}]$ to $3.0 \times 10^{-4}$ $[s^{-1} \text{km}^{-1/2}]$ with a step of $0.05 \times 10^{-4}$ $[s^{-1} \text{km}^{-1/2}]$.

Changing the value of $\kappa$ from 0.4 to 1.0 with a 0.1 step, we estimate $b_k$ ($k=1, 2, 3, 4$) and $F_0$ for each $\kappa$ value. The model parameters which minimize the residual (21) is considered as the best estimates.

The best estimates of the model parameters are as follows. The attenuation coefficients $b_k$ are 0.01, 0.02, 0.03, and 0.04 [s⁻¹] for $f_c = 2, 4, 8, \text{and} 16$ [Hz], respectively. The von Kármán type random-inhomogeneity parameters are $\kappa = 0.8$ and $F_0 = 1.85 \times 10^{-3}$ $[s^{1.02} \text{km}^{-2.01}]$. The value of $F_0$ corresponds to $e^{2.04}/a = 10^{-3.11}$ [km⁻¹] when the average S-wave velocity is set to be $V_0 = 4.2$ [km/s], which well explains the S-wave travel times of our data set. It
should be noted that this method cannot estimate $\varepsilon$ and $a$ independently because the theoretical envelope contains $\varepsilon$ and $a$ as a combination [see (13)].

6. Unified Explanation of Envelope Broadening and Maximum-Amplitude Decay

Figure 10 (a) shows comparisons between the observed envelope broadening and that predicted by the best model of our simulation for each frequency band. The standard deviation of observed data from the theoretically predicted curve is shown at the right bottom of each figure. Figure 10 (b) shows comparisons of maximum-amplitude decay observed and that predicted by the best model of our simulation for each frequency band. Observed values of maximum-amplitude are normalized by the source-energy for each band for each event. Each source-energy is estimated by using the maximum-amplitude decay derived from the theoretical envelope. The standard deviation of observed data from each theoretically predicted curve is shown at the right bottom of each figure. The standard deviations in Figure 10 depend on the theoretical envelope, in other words, we derived those standard deviations considering the envelope broadening and the maximum-amplitude for all the frequency band simultaneously; nevertheless the standard deviations of Figure 10a take almost the same value as those estimated from the independent regression-analyses of the envelope broadening for each frequency band [see Figure 3]. The theoretical envelope successfully gives a unified explanation of the envelope-broadening and maximum-amplitude decay observed at northeastern Honshu. Figure 11 plots the residuals against travel distance for the envelope broadening and the maximum-amplitude decay. The average residuals around 80, 120, 170, 240, and 340km are calculated by dividing the hypocentral distance range into five bins in logarithmic scale. The average values take
almost zero and do not show strong travel-distance dependence in both the envelope broadening and the maximum-amplitude decay.

7. Discussion

PSDF of Fractional S-wave Velocity Fluctuation

We examine how well our estimation constrains the PSDF of fractional S-wave velocity fluctuation. Figure 12 shows the sum of squared residuals $R_\alpha$ [(21)] and $R_s = \sum_{i=1}^{n} R_{s,i}$ [see (20)] against $\kappa$. Parameters $F_0$ and $b_0$ estimated for the corresponding $\kappa$ are listed in Table 4. In Figure 12, both $R_\alpha$ and $R_s$ vary within a few percent in the range of $\kappa$ from 0.5 to 1.0. It means that the envelope duration and the maximum amplitude can be well explained by using any $\kappa$ value ranging from 0.5 to 1.0. For an example, theoretical predictions for the choice of $\kappa = 0.5$ are plotted together with observations in Figure 13. The theoretical predictions can explain the observations as well as the theoretical predictions of $\kappa = 0.8$, although a slight discrepancy can be recognized between the regression line and the theoretical prediction in the figure of envelope broadening at 2Hz band. We note that the regression lines of the envelope duration are independent of theoretical envelope, while those of the maximum amplitude depend on the theoretical envelopes because the source energy is estimated using the theoretical envelope. Hence, the values of $B_M$ and $C_M$ are different between Figure 10 and 13.

Then, we derive the asymptotic representation of the PSDF as a function of wavenumber $m$ as follows. In Figure 14, PSDFs of S-wave velocity fluctuation in the case of $\kappa = 0.8$ are plotted by solid curves. Since our method cannot estimate parameters $\varepsilon$ and $a$ independently, we need to assume $a$ values, which should be larger than a few km [see section 3], to plot the PSDF. Solid curves in Figure 14 correspond to the case of $a = 5, 10$
and 15 km, respectively. The locations of corners in the PSDFs controlled by $a$ are artificial, so that, the lower limit of wavenumber for the meaningful PSDF is the about 0.5 km$^{-1}$ in this study. On one hand, the maximum limit of wavenumber for the meaningful PSDF is considered as twice the wavenumber of the analyzed wavenumber, which is predicted by using the Born approximation [e.g. p.92, Sato and Fehler, 1998]. The corresponding wavenumber 48 km$^{-1}$, twice the maximum wavenumber of wave 24 km$^{-1}$ (16Hz), is shown by a vertical dotted line in Figure 14. In the wavenumber range between 0.5 – 50 km$^3$, the PSDF is well constrained and represented as $P(m) = 0.01 \cdot m^{-0.4} \cdot km^3$ for the case of $\kappa = 0.8$ in spite of the trade-off between $\varepsilon$ and $a$. In a similar way, the PSDFs of $\kappa = 0.5 – 1.0$ are also estimated. The PSDFs are estimated as $P(m) = 0.004 \cdot m^{-5.0} - 0.01 \cdot m^{-4.0} \cdot km^3$ in the wavenumber range between 0.5 – 50 km$^{-1}$. The corresponding area is shaded in Figure 12. In the wavenumber range, the upper limit of the PSDF is given by $0.01 \cdot m^{-3.0} \cdot km^3$ for $\kappa = 0.5$ and the lower limit $0.004 \cdot m^{-5.0} \cdot km^3$ for $\kappa = 1.0$. The PSDF is well constrained around 1.0 km$^{-1}$ although it is not well constrained around 50 km$^{-1}$ which is mainly due to the uncertainty of $\kappa$. This is understandable by considering the scattering coefficient calculated using the Born Approximation [see p. 92, Sato and Fehler, 1998]; the early part of seismograms are mainly composed of small-angle scattered waves which are sensitive to the inhomogeneity spectrum in the lower wavenumbers.

**Amplitude Attenuation**

For the purpose of quantifying seismic wave attenuation, some papers investigate the spectral-amplitude decay against travel distance. Spectral amplitude around maximum amplitude $A_{\phi}(f)$ is often represented as,
where a quality factor $Q_{sp}^{-1}$ is a measure of seismic wave attenuation. However, the interpretation of $Q_{sp}^{-1}$ is ambiguous. Which does it represent intrinsic absorption or scattering attenuation?

The theoretical envelope of the present study can roughly give an physical interpretation of $Q_{sp}^{-1}$ as follows. In the theoretical envelope, the mechanism of the amplitude decay can be decomposed into three factors; the geometrical spreading factor $r^{-1}$, an attenuation factor $t_{sa}^{-3/2}$ which is considered as the scattering attenuation due to small-angle scattering [see Figure 7], and an exponential attenuation factor $e^{-b/2}$ which is considered as the sum of the intrinsic absorption and the scattering attenuation due to the large-angle scattering. When we consider no intrinsic absorption and no scattering loss due to large-angle scattering, i.e. $b = 0 \, \text{s}^{-1}$, the value of integrated squared-amplitude over time with the correction of geometrical spreading is independent of travel distance [see p.248 Sato and Fehler, 1998; Saito et al., 2002a]. In other words, integrating squared-amplitude over time corresponds to the correction of the scattering attenuation due to small-angle scattering. Therefore, spectral amplitude $A_{sp}(f)$, which is also estimated by the integration over a time window around maximum amplitude, can be considered as the amplitude corrected by the scattering attenuation due to small-angle scattering. From those consideration, the quality factor $Q_{sp}^{-1}$ estimated by the spectral-amplitude decay can be considered as the contribution from intrinsic absorption and scattering loss due to large-angle scattering; we expect that the value

$$A_{sp} \approx \frac{1}{r} \exp \left[ -\frac{\pi fr}{V_0 Q_{sp}^{-1}} \right], \quad (22)$$
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\[ Q_b^{-1} = \frac{b}{2\pi f} \]  

(23)

would be the same as \( Q_p^{-1} \). Here, the time window for the estimation of spectral amplitude should be chosen according to travel distance in order that the time window should be larger than the envelope duration.

The quality factor \( Q_p^{-1} \) in the forearc side in northeastern Honshu is recently estimated by Takahashi et al. [2004] from the analysis of the spectral-amplitude decay. They select the 225 events of which hypocentral distance ranges about 50 – 170 km and analyze the seismograms observed at Hi-net stations. The time window for the calculation of spectral amplitude is 6 [s]. It corresponds to the duration \( t_q \) at travel distance about 180km when we use the relation (1) with the estimated regression coefficients. Figure 15 compares the \( Q_b^{-1} \) estimated in this study and \( Q_p^{-1} \) estimated by Takahashi et al. [2004]. We estimate the error bars of \( Q_b^{-1} \) from our data-set by using the bootstrap method changing the range of \( \kappa \) from 0.5 to 1.0. The error bars of the estimation in Takahashi et al. [2004] are larger than those of our study. This is mainly due to the travel distance range of the analyzed events; we analyzed the events located in the hypocentral-distance range between 75 - 410km which is wider than Takahashi et al. [2004], so that our estimation is more stable. As the theoretical prediction derived above, the values of \( Q_b^{-1} \) and \( Q_p^{-1} \) are in good agreement within the range of error bars even though those values are estimated by different methods.

Large-angle Scattering and Coda Excitation

Although the purpose of the present study is the modeling of the envelope duration and the maximum amplitude, coda, which is the later part of envelope (usually later than twice the S-wave travel-time), has also been considered as one of the most important features in
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high-frequency seismograms. The coda excitation has been usually parameterized by total scattering coefficient $g_\alpha$ based on the radiative transfer theory with the assumption of isotropic scattering; the $g_\alpha$ values estimated in past studies are about $10^{-2.5} - 10^{-1.4}$ km$^{-1}$ in the frequency range 1 - 20 Hz [e.g. Figure 3.10, Sato and Fehler, 1998]. Here, we try to discuss the coda excitation by comparing the $g_\alpha$ value of the past studies and the predictions from our study.

The coda is composed of the scattered waves; the large-angle scattering attenuates the wave energy around the maximum amplitude and the attenuated energy appears again in seismograms as coda. Hence, the scattering coefficient due to the large-angle scattering is comparable to the total scattering coefficient of the isotropic scattering, as a measure of the coda excitation. In this study, the large-angle scattering is introduced as the attenuation represented by the parameters $b_\alpha$ to the theoretical envelope. If we assume no intrinsic absorption, the attenuation factor $\exp(-bt/2)$ can be considered as the attenuation due to large-angle scattering as $\exp(-g_L V_\alpha t/2)$ where $g_L$ is the scattering coefficient due to the large-angle scattering. The value of $g_L$ calculated from the estimated $b_\alpha$ is $g_L = 10^{-2.6} - 10^{-2.0}$ km$^{-1}$. This value is in the same order as the total scattering coefficient $g_\alpha = 10^{-2.5} - 10^{-1.4}$ km$^{-1}$ reported in past studies. It may suggest that envelope broadening, maximum-amplitude decay, and coda excitation can be explained simultaneously by the scattering of wave without intrinsic absorption. However, in case the contribution of the intrinsic absorption is large, coda is not excited enough.

On the other hand, independently from the above method, the coda excitation can be predicted from the PSDF of fractional wave velocity fluctuation by the following method [Gusev and Abubakirov, 1996; Saito et al. 2003; Sato et al., 2004]. The scattering coefficient is estimated from the PSDF by using the Born approximation. Momentum transfer scattering
coefficient $g_m$ defined by using the scattering coefficient works as the total scattering coefficient in the radiative transfer theory with isotropic scattering [p. 188, Morse and Feshbach, 1953]. The value of $g_m$ calculated from the PSDF estimated in this study is $g_m \approx 10^{-2.6} - 10^{-2.3}$ km$^{-1}$. This is smaller than the total scattering coefficient reported in past studies. It means that the PSDF estimated from the envelope duration and peak amplitude cannot make large enough coda excitation. However, we note that elastic-wave scattering should be considered for rigorous estimation of an effective total scattering coefficient because the conversion scattering is important in the later part of the envelope.

From the above discussions, there is a possibility that the model parameters estimated in this study cannot generate large enough coda waves. We expect the joint analysis of the coda excitation in addition to the envelope broadening and the maximum-amplitude decay to clarify the relation among them and estimate reasonable model parameters. To accomplish this task, the envelope simulation method which accurately predicts the envelope from onset to coda is necessary. We developed the method in the 2-D scalar wave case [Saito et al., 2003; Sato et al., 2004]. It will be necessary to extend the method to elastic-wave scattering and 3-D wave propagation for the application to observed seismograms.

For More Precise Estimation

For more precise estimation of the lithosphere, following points should be developed. To constrain the PSDF of the velocity fluctuation well, the analysis of the coda excitation would be useful. The coda excitation is sensitive to short-wavelength component of the inhomogeneity [e.g. Saito et al. 2003]. Making use of this feature, we would be able to constrain the PSDF in large-wavenumber which are not well constrained in the present study [see Figure 14]. Suitable parameterization of envelopes is also necessary. We measured $t_q$ values from a number of observed envelopes and compare them with those of theoretical
ensemble-average envelopes, where we consider the average of $t_q$ over many traces as the $t_q$ value of the ensemble average-envelope. But this assumption has not been verified yet. The same applies to $A_{\text{max}}$. To ensure this or find more suitable parameterization, the relation between the ensemble-average envelope and each single-realized envelope must be investigated. It is also important to employ more realistic and general structure such as localized inhomogeneity, anisotropic inhomogeneity, depth dependent velocity structure, etc. Especially, localized inhomogeneity and anisotropic inhomogeneity would significantly affect the envelope [e.g. Gusev and Abubakirov, 1999a, 1999b; Ryberg et al. 2000]. We should develop the envelope simulation method under those realistic assumptions and apply that to observed envelopes.
6. Conclusions

We modeled the envelope broadening and the maximum-amplitude decay simultaneously by the simulation using the Markov approximation for wave propagation through random media. The early part of S-wave envelopes observed in the forearc side of the volcanic front in northeastern Honshu, Japan has following characteristics in the frequency range from 2 to 16Hz in the distance range from 75 to 410 km: the envelope duration increases with increasing travel distance in proportion to distance to the power 1.7 to 1.8 and the maximum amplitude decreases with increasing travel distance in proportion to distance to the power –3 to –2. These observations are explained by the small-angle scattering of waves around the forward direction in the media having randomly fluctuating S-wave velocity with frequency dependent attenuation. The PSDF of the wave-velocity fluctuation represented by the von Kármán-type random media is estimated as $P(m) = 0.004 \cdot m^{-5.0} - 0.01 \cdot m^{-4.0}$ km$^3$ in the wavenumber $m$ between 0.5 – 50 km$^{-1}$. The joint analysis of envelope broadening and maximum-amplitude decay is superior in estimating inhomogeneous velocity structure and attenuation structure over the analysis of envelope broadening or maximum-amplitude decay alone. This study indicates that the envelope simulation method using the Markov approximation is useful in the comprehensive understanding of the duration and the maximum amplitude based on a wave scattering theory.
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References


Figure Captions

Figure 1. (a) Map view of the location of Hi-net stations (diamonds) and epicenters of analyzed events (stars) in northeastern Honshu, Japan. Volcanoes and the volcanic front are shown by triangles and a bold dashed line, respectively. (b) EW vertical cross-section of the hypocenters of analyzed events (stars). A thin dashed line shows the upper bound of the subducted Pacific plate.

Figure 2. (a) Example of NS component velocity-seismogram observed at a Hi-net station (KJSH) for earthquake #5 and (b) root-mean-square (RMS) envelope of the 8Hz band. A fine dashed line indicates the S-wave onset. A coarse vertical dashed line indicates the time when the amplitude decrease to the half of the maximum amplitude. The observed RMS envelope is characterized by the maximum amplitude $A_{max}$ and the envelope duration $t_q$.

Figure 3. Log-log plots of envelope-duration $t_q$ (dot) against hypocentral distance for four frequency bands. Dashed lines are linear regression lines given by equation (1). Regression coefficients $B_q$, $C_q$ and the standard deviation of observed data from each regression line are shown at the bottom of each figure.

Figure 4. Log-log plots of maximum-amplitude (dot) against hypocentral distance for the 8Hz band for eight events. Lines are linear regression lines given by equation (2). Regression coefficient $C_M$ for each event is shown at the left of each regression line.

Figure 5. 3-D random media of von Kármán type. (a) The power spectral density function (PSDF) of fractional velocity fluctuation in 3D space for various values of $\kappa$. (b) Example 2-D section of von Kármán type random media.

Figure 6. Coordinates used in the formulation. At a large distance $r$ from the source, we take the local Cartesian coordinates, where the one axis is in the direction of a receiver and the other two axes are in the transverse plane which is tangent to the sphere of radius $r$.

Figure 7. Theoretical mean-square envelopes calculated by using the Markov approximation. Mean-square amplitude are scaled by using the characteristic time $t_M$ and geometrical
spreading factor. Reduced time is scaled by $t_u$ [see Table 3 for the correspondence between $p$ and $\kappa$].

Figure 8. Theoretical root-mean-square envelopes of waves with center frequency 8Hz in von Kármán-type random media ($V_o = 4$km/s, $\kappa = 0.8$, $\varepsilon = 0.05$, $a = 5$km) calculated by using the Markov approximation. Envelopes calculated at distances 100, 150, 200, 250, and 300km from a source are plotted together. A broken curve shows the maximum-amplitude decay due to geometrical spreading which is proportional to the reciprocal of hypocentral distance.

Figure 9. Contour plots of the residual value with respect to $b$ and $F_0$ in the 16 Hz band: (a) the plot for the envelope duration [eq. (14)]; (b) the plot for the maximum amplitude [eq. (15)]; (c) the plot for both envelope duration and maximum amplitude [eq. (16)]. In Figure (a) and (b), the area with residuals smaller than 1.05 is shaded by dark gray and the area with residuals smaller than 1.10 is shaded by light gray. In Figure (c), the area with residuals smaller than 2.10 is shaded by dark gray and the area with residuals smaller than 2.20 is shaded by light gray.

Figure 10. (a) Log-log plots of the envelope duration $t_v$ against hypocentral distance and (b) log-log plots of the maximum amplitude $A_{\text{max}}$. Dots are observed values and gray dashed lines are the regression lines given by (1) or (2). The regression coefficients and the standard deviation of observed data from each theoretically predicted curve are shown at the bottom. Solid curves are theoretically predicted by the envelope simulation using the parameters $\kappa = 0.8$, $\varepsilon^{2.04}/a = 10^{-3.11}$km$^{-1}$, $b = 0.01, 0.02, 0.03$ and 0.04 s$^{-1}$ for 2, 4, 8 and 16Hz band, respectively:

Figure 11. (a) Residuals between observed and calculated $\log t_v$ values against hypocentral distance. (b) Residuals between observed and calculated $\log A_{\text{max}}$ values against hypocentral distance. Each residual is plotted by Gray dot. The average residuals and the standard deviations around 80, 120, 170, 240, and 340km are plotted by squares and bars.
Figure 12. The variation of the sum of squared residuals against $\kappa$. The value of $R_q$ [see (21)] is plotted by closed circles with a solid line, where the values is normalized by the minimum value. The value of $R_j = \sum_{i=1}^{4} R_{j,i}$ [see (20)] against various values of $\kappa$ is plotted by open circles with a broken line, where the value is normalized by the minimum value.

Figure 13. (a) Log-log plots of the envelope duration $\tau_q$ against hypocentral distance and (b) log-log plots of the maximum amplitude $A_{\max}$. Dots are observed values and gray dashed lines are the regression lines given by (1) or (2). The regression coefficients and the standard deviation of observed data from each theoretically predicted curve are shown at the bottom. Solid curves are theoretically predicted by the envelope simulation using the parameters $\kappa = 0.5$, $e^{2 \phi/a} = 10^{-3.87}$ km$^{-1}$, $b = 0.00, 0.01, 0.02$ and $0.03$ s$^{-1}$ for 2, 4, 8 and 16Hz band, respectively.

Figure 14. Log-log plots of von Kármán-type PSDFs estimated in northeastern Honshu, Japan. Solid curves show the PSDFs of $(\kappa, \varepsilon, a) = (0.8, 0.07, 5km)$, $(0.8, 0.09, 10km)$, and $(0.8, 0.11, 15km)$. Shaded area shows the range of the PSDF with the variation of $\kappa$ from 0.5 to 1.0. The wavenumber range indicated by an arrow shows the range of analyzed wavenumber. A vertical dashed line shows the twice the maximum analyzed-wavenumber.

Figure 15. Frequency dependence of quality factors estimated in northeastern Honshu, Japan. The quality factors defined using the attenuation coefficients [(23)] are plotted by closed circles with error bars. The quality factors estimated from the spectral-amplitude attenuation against travel distance are plotted by squares (northern part of N.E. Honshu) and triangles (southern part of N.E. Honshu) with error bars [Takahashi et al., 2004].
### Table 1. Event List Used for the Analysis

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### Table 2. Parameters for the Power-Law Relation Between Maximum Amplitude and Hypocentral Distance  \( A_{\text{max}} \propto r^{C_M} \)

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<th>Average Distance [km]</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>–2.34</td>
<td>–3.16</td>
<td>–3.62</td>
<td>–4.23</td>
<td>79 - 336</td>
<td>177</td>
</tr>
<tr>
<td>#2</td>
<td>–1.99</td>
<td>–1.82</td>
<td>–2.25</td>
<td>–2.63</td>
<td>90 - 393</td>
<td>217</td>
</tr>
<tr>
<td>#3</td>
<td>–2.97</td>
<td>–3.43</td>
<td>–3.47</td>
<td>–3.81</td>
<td>94 - 262</td>
<td>176</td>
</tr>
<tr>
<td>#4</td>
<td>–1.84</td>
<td>–1.83</td>
<td>–2.53</td>
<td>–3.01</td>
<td>76 - 408</td>
<td>200</td>
</tr>
<tr>
<td>#5</td>
<td>–1.99</td>
<td>–1.80</td>
<td>–2.08</td>
<td>–2.11</td>
<td>120 - 365</td>
<td>206</td>
</tr>
<tr>
<td>#6</td>
<td>–3.11</td>
<td>–3.31</td>
<td>–3.34</td>
<td>–3.83</td>
<td>108 - 330</td>
<td>190</td>
</tr>
<tr>
<td>#7</td>
<td>–2.18</td>
<td>–2.75</td>
<td>–2.76</td>
<td>–3.03</td>
<td>90 - 362</td>
<td>163</td>
</tr>
<tr>
<td>#8</td>
<td>–1.53</td>
<td>–2.00</td>
<td>–1.93</td>
<td>–2.68</td>
<td>119 - 311</td>
<td>176</td>
</tr>
<tr>
<td>Average</td>
<td>–2.2</td>
<td>–2.5</td>
<td>–2.7</td>
<td>–3.2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3. Numerical Estimation of Parameters $C$ and $p$ for Different Values of $\kappa$ for von Kármán-type Random Media [After Saito et al., 2002a].

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>$C (\kappa)$</th>
<th>$p (\kappa)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.56</td>
<td>1.19</td>
</tr>
<tr>
<td>0.2</td>
<td>1.06</td>
<td>1.38</td>
</tr>
<tr>
<td>0.3</td>
<td>1.56</td>
<td>1.56</td>
</tr>
<tr>
<td>0.4</td>
<td>2.00</td>
<td>1.71</td>
</tr>
<tr>
<td>0.5</td>
<td>2.28</td>
<td>1.83</td>
</tr>
<tr>
<td>0.6</td>
<td>2.31</td>
<td>1.91</td>
</tr>
<tr>
<td>0.7</td>
<td>2.14</td>
<td>1.95</td>
</tr>
<tr>
<td>0.8</td>
<td>1.90</td>
<td>1.98</td>
</tr>
<tr>
<td>0.9</td>
<td>1.68</td>
<td>1.99</td>
</tr>
<tr>
<td>1.0</td>
<td>1.50</td>
<td>1.99</td>
</tr>
</tbody>
</table>

Table 4. Estimation of the Parameters of the von Kármán-type Random Inhomogeneity and Attenuation for Different Values of $\kappa$.

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>$F_0$ [s$^{-1+4/p(\kappa)}$ km$^{-1-2/p(\kappa)}$]</th>
<th>$\epsilon^{2[p(\kappa)-1]}a^{-1}$ [km$^{-1}$]</th>
<th>$b$ [s$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>$0.30 \times 10^{-4}$</td>
<td>$10^{-0.50}$</td>
<td>0.01</td>
</tr>
<tr>
<td>0.5</td>
<td>$0.70 \times 10^{-4}$</td>
<td>$10^{-0.87}$</td>
<td>0.01</td>
</tr>
<tr>
<td>0.6</td>
<td>$1.20 \times 10^{-4}$</td>
<td>$10^{-0.48}$</td>
<td>0.01</td>
</tr>
<tr>
<td>0.7</td>
<td>$1.55 \times 10^{-4}$</td>
<td>$10^{-0.28}$</td>
<td>0.01</td>
</tr>
<tr>
<td>0.8</td>
<td>$1.85 \times 10^{-4}$</td>
<td>$10^{-0.11}$</td>
<td>0.01</td>
</tr>
<tr>
<td>0.9</td>
<td>$1.95 \times 10^{-4}$</td>
<td>$10^{-0.02}$</td>
<td>0.01</td>
</tr>
<tr>
<td>1.0</td>
<td>$1.95 \times 10^{-4}$</td>
<td>$10^{-2.97}$</td>
<td>0.01</td>
</tr>
</tbody>
</table>
Figure 1
Figure 2
Figure 3
Figure 4
Figure 5
Transverse Plane

Source

Receiver

Figure 6
Figure 7
Figure 8
Figure 9
κ = 0.8

Figure 10
Figure 11
Figure 12
\( \kappa = 0.5 \)

(a) Hypocentral Distance [km]

(b) Hypocentral Distance [km]

Figure 13
Figure 14
Figure 15