SEISMOGRAM ENVELOPE INVERSION FOR HIGH-FREQUENCY SEISMIC ENERGY RADIATION FROM MODERATE-TO-LARGE EARTHQUAKES

Hisashi Nakahara

Abstract

Studies of high-frequency (above 1 Hz) earthquake source processes are important not only to clarify the earthquake source process on smaller length scales but also to quantitatively predict strong ground motion. However, the application of conventional waveform inversion methods is not straightforward for high frequencies, because random heterogeneities in the Earth cause incoherent scattered waves and the source process is also hard to treat deterministically. To obviate these difficulties, seismogram envelope inversion methods have been developed since the 1990s for clarifying high-frequency earthquake source processes. In this chapter, we first give a broad discussion of the methods in terms of data types, Green’s function, source parameters, inversion methods, and so on. We developed an envelope inversion method in 1998, in which we used theoretical envelope Green’s functions based on the radiative transfer theory as a propagator from a source to a receiver, and estimated the spatial distribution of high-frequency seismic energy radiation from an earthquake fault plane. We have applied the envelope inversion method to nine moderate-to-large earthquakes. Here, we compile the results and clarify some characteristics of high-frequency seismic energy radiation from moderate-to-large earthquakes. Concerning a scaling of high-frequency radiated energy, logarithm of the high-frequency seismic energy is found to be proportional to the moment magnitude with a coefficient of proportionality of 1, which is different from 1.5 for whole-band seismic energy. Moreover, a regional difference in high-frequency seismic energy radiation is detected for the earthquakes analyzed: Earthquakes in offshore regions of northeastern Japan are found to be more energetic by about an order of magnitude than inland earthquakes in Japan and Taiwan. Regarding the spatial relations, we find four earthquakes in which high-frequency radiation occurs dominantly at the edges of asperities (areas of large fault slip); in four cases there is no correlation between locations of high-frequency radiation and asperities. For one earthquake, we have no fault slip model. So far, reasons for the variation are not known yet, heterogeneous distribution of stress, strength, and material properties may control the variability. These characteristics will provide important information for the study of high-frequency earthquake source process and improvements for predicting strong ground motion.

Key Words: Envelope inversion, high-frequency seismic energy, prediction of strong ground motion. © 2008 Elsevier Inc.
1. Introduction

Studies on high-frequency (usually higher than 1 Hz) earthquake source processes of moderate-to-large earthquakes are important not only for clarifying the earthquake source processes in detail but also for quantitatively predicting strong ground motion. However, it is not easy to apply conventional waveform inversion methods for high frequencies. One of the reasons is that random heterogeneities in the Earth cause scattering and produce incoherent wave trains in seismograms (Sato and Fehler, 1998). Another reason is that the source process of moderate-to-large earthquakes becomes complex at higher frequencies so it is hard to treat it deterministically (e.g., Koyama, 1994).

Historically, various approaches have been developed to clarify high-frequency earthquake source process of moderate-to-large earthquakes. Hypocenter determination of subevents may be the most primitive method, in which arrival times of large-amplitude phases are picked on strong-motion records and their source locations are determined. If the large-amplitude phases have not originated from structures (e.g., reflected or scattered or refracted waves), the phases are attributed to earthquake sources. Waveform data other than these large-amplitude phases are neglected in this kind of analysis. Moreover, information on the amplitude of the phases is not considered. Measurements of earthquake source spectra had been conducted and led to heterogeneous fault rupture models (e.g., Gusev, 1983; Papageorgiou and Aki, 1983; Koyama, 1985). This kind of study uses amplitude information of all seismograms but lacks spatial resolution, because the amplitude source spectrum is calculated from seismograms in a long time window. To improve the spatial resolution, it is necessary to investigate the temporal change in observed signals. So, time series of seismogram envelopes have been used since the 1990s to invert for high-frequency earthquake source processes. (e.g., Gusev and Pavlov, 1991; Cocco and Boatwright, 1993; Zeng et al., 1993; Kakehi and Irikura, 1996).

In particular, Zeng et al. (1993) and Kakehi and Irikura (1996) have succeeded in clarifying the spatial distribution of the intensity of high-frequency wave radiation on earthquake fault planes. They have also enabled the comparison of the results to the spatial distribution of fault slip obtained by conventional waveform inversions in lower frequency ranges. The relationship between the locations of high-frequency wave radiation and the locations of asperities (regions with large fault slip) identified by studying low-frequency data is an important characteristic to be explained by the theory of dynamic earthquake rupture. At the same time, this relation is recently one of the central issues in strong-motion seismology from a perspective of the prediction of broadband strong ground motion (e.g., Irikura and Miyake, 2001). Regarding the lower frequencies, fault slip models found by waveform inversion methods have been accumulating since the middle of 1980s. Based on the slip models, some statistical characteristics in slip distributions on earthquake faults have been successfully extracted (e.g., Somerville et al., 1999; Mai and Beroza, 2000). On the contrary, the number of high-frequency envelope inversion analyses is much smaller than that of waveform inversion analyses in lower frequencies. Accordingly, our knowledge about the high-frequency seismic wave radiation is much smaller than that about fault slip models. However, the number of high-frequency envelope inversion analyses has been gradually increasing since the 1990s. Therefore, we here make the first trial to extract statistical characteristics in high-frequency wave radiation based on the previous results obtained by envelope inversions. Moreover, we discuss the relationship between the locations of high-frequency wave
radiation and those of low-frequency wave radiation. The compilation will provide us with important information which should be incorporated into the simulation of broadband strong ground motion.

2. Envelope Inversion Methods

2.1. General Framework

As far as the Earth can be assumed to be a linear system, an observed seismogram \( u(t) \) can be expressed by the convolution between a source time function \( S(t) \), an impulse response for propagation paths \( P(t) \), that is the Green’s function, and a receiver (site) response function \( R(t) \) as

\[
 u(t) = S(t)^* P(t)^* R(t), \tag{1}
\]

where \(^*\) means convolution. If each of the time functions is a quasi-stationary, mutually uncorrelated, and narrowband random signal with zero mean, the similar convolution relation may be valid for the instantaneous power or the envelope of \( u(t) \) as

\[
 <u(t)^2> = <S(t)^2>^* <P(t)^2>^* <R(t)^2>, \tag{2}
\]

where \(<>\) means an ensemble average. For more detailed explanation, please refer to Chapter 5 in Ishimaru (1978). A schematic illustration is shown in Fig. 1. Practically, the

![Diagram](https://via.placeholder.com/150)

**Fig. 1.** Schematic illustration of the envelope inversion method of Nakahara *et al.* (1998). Seismic energy radiated from a double-couple source located at the center of the \( k \)th subfault on an earthquake fault (gray parallelogram) is multiply scattered, amplified beneath the \( i \)th receiver (solid triangle) and reaches the receiver at the \( j \)th time. Gray circles represent point-like isotropic scatterers for S waves randomly distributed in a three-dimensional space. The background S-wave velocity structure is assumed to be homogeneous.
ensemble average is substituted by a moving time average. Therefore, \(<u(t)^2>\) can be calculated as a mean-squared (MS) envelope or a squared envelope of the analytic signal of the observed seismogram \(u(t)\). Assuming that high-frequency seismic waves are incoherent, we can calculate the observed envelope by the convolution of each envelope for the source, the propagation path, and the site effects, respectively. This characteristic facilitates a direct calculation of an envelope of Green’s function, which we call the envelope Green’s function.

We note that spatial coordinates do not appear explicitly in Eqs. (1) and (2). So, the equations are for a point source and a receiver in a strict sense. For multiple sources and finite-sized faults, an additional convolution with respect to the spatial coordinates is necessary. Such an extension is straightforward for Eq. (1) because of the superposition principle for linear systems. However, the extension of Eq. (2) has to rely on an assumption that energy radiated from different subsources is additive. This assumption is equivalent to the incoherence of high-frequency seismic waves.

Most envelope inversion methods are based on the finite-fault version of Eq. (2). However, some others make a forward calculation of synthetic seismograms based on the finite-fault version of Eq. (1) and the envelope is used just in fitting for inversion. However, differences between the approaches are small because signals are assumed to be random with zero mean.

2.2. A Classification of Current Envelope Inversion Methods

It may be helpful to find similarities and differences in the envelope inversion methods which have been proposed to date. Here, we make a classification of the methods in terms of data types, frequency ranges, source parameters, Green’s functions, inversion methods, and so on as shown in Table 1. The source is modeled as a point source, multiple point sources, or finite-sized faults. Frequency ranges are higher than 0.45 Hz in all the methods. Types of the Green’s function are empirical or theoretical, and the data are seismograms or envelopes. Estimation of source parameters is conducted by trial-and-error methods, inversion methods, or deconvolution methods.

Iida and Hakuno (1984) is a pioneering paper in using temporal change in absolute amplitude of acceleration seismograms for estimating intensity of source radiation on earthquake fault planes. Because the amount of available data was small, trial-and-error modeling was conducted for the 1968 Tokachi-Oki, Japan, earthquake and 1978 Miyagi-Ken-Oki, Japan, earthquake. Gusev and Pavlov (1991) performed deconvolution of MS envelopes of far-field P-wave velocity seismograms of the 1978 off Miyagi, Japan, earthquake (M 7.6), and estimated the location of a “short-period radiator,” which corresponds to a centroid for high-frequency wave radiation. Cocco and Boatwright (1993) deconvolved MS envelopes of acceleration records and estimated the power rate function for an after-shock (M<sub>L</sub> 5.9) of the 1976 Friuli earthquake. Kakehi and Irikura (1996) estimated high-frequency wave radiation areas on the fault of the 1993 Kushiro-Oki earthquake (M<sub>W</sub> 7.6) by using root MS envelopes of acceleration seismograms. These four studies used seismograms of small earthquakes as empirical Green’s functions. When there are records of small events available which have the same location and focal mechanism as a target large event (mainshock), realistic propagation effects can be naturally included in the empirical Green’s function. However, that is not always the case. Petukhin et al. (2004) relaxed this constraint a little and used average envelopes of small events located in the same area as a mainshock as the envelope Green’s function, and inverted MS envelopes of squared velocity records of
<table>
<thead>
<tr>
<th>References</th>
<th>Data type</th>
<th>Frequency (Hz)</th>
<th>Source model</th>
<th>Source parameters</th>
<th>Green’s function</th>
<th>Estimation methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gusev and Pavlov (1991)</td>
<td>Teleseismic sqr. vel.</td>
<td>0.45–1.75</td>
<td>Point source</td>
<td>Seismic energy</td>
<td>Empirical envelope</td>
<td>Deconvolution and inversion</td>
</tr>
<tr>
<td>Zeng et al. (1993)</td>
<td>Near-field sqr. disp.</td>
<td>&gt;5</td>
<td>Finite fault</td>
<td>Displacement</td>
<td>Ray theory envelope</td>
<td>Inversion</td>
</tr>
<tr>
<td>Nakahara et al. (1998)</td>
<td>Near-field S-wave sqr.</td>
<td>1–16</td>
<td>Finite fault</td>
<td>Seismic energy</td>
<td>Radiative transfer theory envelope</td>
<td>Inversion</td>
</tr>
</tbody>
</table>
the 1992 Avachinsky Gulf earthquake (Mw 6.8). Regarding the empirical Green’s function method, estimated source parameters are relative to those of a small event for which the empirical Green’s function is used.

The use of theoretical Green’s function is superior in terms of estimating absolute values of source parameters as far as a reference station is correctly selected. However, the number of the studies using theoretical envelope Green’s functions is very small. Zeng et al. (1993) inverted MS envelopes of displacement seismograms from the 1989 Loma Prieta earthquake by using ray-theoretically calculated Green’s functions, and mapped the high-frequency source radiation intensity on the earthquake fault. Nakahara et al. (1998) inverted MS envelopes of squared velocity seismograms using the radiative transfer-based theoretical envelope Green’s function, and estimated the spatial distribution of high-frequency seismic energy radiation on the fault plane of the 1994 far east off Sanriku earthquake (Mw 7.7).

Finally, it is worth while to refer to two pioneering studies dealing with envelopes though they are not inversion studies in a strict sense. Midorikawa and Kobayashi (1979) proposed a method to calculate a velocity response spectrum on seismic bedrock due to the rupture of a finite-sized fault. The method estimates an average empirical envelope of a velocity motion of an oscillator with a certain natural period on the seismic bedrock due to the rupture of a subfault. The envelope of the oscillator due to the entire fault is obtained by summing up contributions from all subfaults. The maximum amplitude of the envelope corresponds to the velocity response at the period on the seismic bedrock. The method serves for a forward modeling of strong ground motion based on a fault model. Koyama and Zheng (1985) proposed a technique to estimate spectral amplitude at a frequency of about 1 Hz from envelopes of teleseismic P-wave displacement records obtained by short-period sensors of the World Wide Standardized Seismic Network. After the correction of propagation effects and instrumental responses, they estimated the short-period seismic moment for 79 large earthquakes, and verified that the radiation of high-frequency seismic waves is incoherent.

2.3. The Method of Nakahara et al. (1998)

Here, we make a brief explanation of the envelope inversion method of Nakahara et al. (1998), because we will be mainly concerned with the results based on the method. The method uses the theoretical envelope Green’s function developed by Sato et al. (1997) based on the radiative transfer theory (e.g., Chandrasekhar, 1960). The radiative transfer theory, sometimes called the energy transport theory or the multiple scattering theory, was first introduced to seismology by Wu (1985) for stationary cases. Later, it was extended to time-dependent cases numerically by Gusev and Abubakirov (1987) and theoretically by Zeng et al. (1991), because seismic energy is usually radiated from a transient (approximately impulsive) earthquake source. Because the radiative transfer theory was initially used to explain coda envelopes of local earthquakes, the source radiation pattern, which has a large effect on early parts but a small effect on later coda parts, had not been included in the modeling. Sato et al. (1997) succeeded in introducing the radiation pattern for a point shear dislocation (double couple) source and enabled the synthesis of S-wave seismogram envelopes from the direct waves through coda. The study paved a way to the application of the radiative transfer theory to detailed earthquake source studies.
Here, we briefly explain the envelope Green’s function used in our envelope inversion method. As shown in Fig 2(a), point-like isotropic scatterers are assumed to be distributed randomly and homogeneously in an infinite medium, where the background S-wave velocity $V$ is constant. Only S waves are considered in the modeling. A double-couple source is located at the origin from which seismic energy of unit amplitude is impulsively radiated. In the framework of the radiative transfer theory, the energy density $E_G(x, t)$ at a location $x = (r, \theta, \phi)$ in a spherical coordinate system and time $t$ can be expressed by the following integral equation:

$$E_G(x, t) = R(\theta, \phi)G(x, t) + g_0 V \int \int dx' \int_{-\infty}^{\infty} d\tau' G(x - x', t - \tau') E_G(x', \tau'),$$  \hspace{1cm} (3)

where $r = |x|$, $\theta$ is the zenith angle and $\phi$ is the azimuth angle, and $g_0$ is the total scattering coefficient characterizing the scattering power per unit volume. The function $E_G$ is the envelope Green’s function in the scattering medium. $R(\theta, \phi)$ is the radiation pattern of S-wave energy which is normalized as $\int R(\theta, \phi) d\Omega = 4\pi$. The first term in the right-hand side means the coherent part corresponding to the direct wave. The second term means the scattered energy, which is given by integrating the contributions from the last scattering point $x_0$ and at lapse time $\tau'$. The propagator function $G(x, t)$ is expressed as

$$G(x, t) = \exp\left(-\frac{g_0 V + \eta_0 t}{4\pi V r^2}\right) \delta\left(t - \frac{r}{V}\right) \text{ for } t \geq 0.$$  \hspace{1cm} (4)

This is characterized by geometrical spreading, time lag due to propagation, and exponential decay due to intrinsic absorption and scattering attenuation of seismic energy. Intrinsic absorption $\eta$ is related to the intrinsic $Q$ value as $Q^{-1} = \eta/\omega$ for an angular frequency $\omega$.

For a double-couple source with the fault normal vector in the first axis and slip vector in the second axis as shown in Fig. 2(a), we can explicitly express $R(\theta, \phi)$ in terms of the spherical harmonics with the order $n$ of up to four as

$$R(\theta, \phi) = \sum_{n=0,2,4} \sum_{m=-n}^{n} a_{nm}Y_{nm}(\theta, \phi) = \sqrt{4\pi} Y_{0,0}(\theta, \phi) + \frac{5}{7} \sqrt{\frac{4\pi}{5}} Y_{2,0}(\theta, \phi) - \frac{2}{7} \sqrt{\frac{4\pi}{9}} Y_{4,0}(\theta, \phi) + \frac{\sqrt{280\pi}}{21} \left( Y_{4,4}(\theta, \phi) + Y_{4,-4}(\theta, \phi) \right),$$  \hspace{1cm} (5)

where $\theta$ is measured from the null axis (the third axis), and $\phi$ is measured from the fault normal (the first axis).

We solve Eqs. (3–5) for the envelope Green’s function $E_G(x, t)$ by using the Fourier transform in space, the Laplace transform in time, and the spherical harmonics expansion with respect to radiation angles. The energy density can be written in a spherical harmonics expansion with the expansion coefficients of the radiation pattern:

$$E_G(x, t) = \sum_{n=0}^{\infty} E_{G,n}(r, t) \sum_{m=-n}^{n} a_{nm} Y_{nm}(\theta, \phi),$$  \hspace{1cm} (6)
Fig. 2. (a) Configuration of a double-couple source with the fault normal vector in $x_3$ axis and the slip vector in $x_2$ axis. (b) Theoretical envelopes calculated for the double-couple source shown in (a) (solid curves) and those for an isotropic source (broken curves). Horizontal axis is normalized time and vertical axis is normalized energy density. Envelopes at three different receivers are shown from top to bottom. Although early parts of envelopes clearly exhibit a difference in energy density due to the radiation pattern, the difference becomes smaller and smaller as time elapses.
where

\[
E_{G,n}(r, t) = \frac{e^{-(g_0 V + \eta)t}}{4\pi r^2 V} \delta(t - \frac{r}{V}) H(t) + \frac{g_0 e^{-(g_0 V + \eta)t}}{4\pi V t} Q_n \left( \frac{(Vt/r)^2 + 1}{2(Vt/r)} \right) H \left( t - \frac{r}{V} \right) \\
+ \frac{g_0^2 V^2}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{e^{i\omega t}}{2\pi} \int_{-\infty}^{\infty} dk \frac{e^{ikr}}{2\pi} k u_n(kr) \frac{\overline{G}_n(k, i\omega) \overline{G}_n^*(k, i\omega)}{1 - g_0 V G_0(k, i\omega)}.
\]

(7)

The first term is the coherent term, the second is the single scattering term, and the third is multiple scattering term with the order higher than or equal to 2. \( Q_n \) is the Legendre polynomial of the second kind. Function \( u_n(x) \) originates from the spherical Bessel function and is defined as

\[
u_n(x) \equiv \sum_{s=0}^{n} \frac{i^n (n+s)!}{s!(n-s)!(2x)^s}.
\]

(8)

Function \( \overline{G}_n(k, s) \) corresponds to the Laplace transform of the spherical Bessel function:

\[
\overline{G}_n(k, s) = \frac{1}{kV} \left( \frac{kV}{2(s + gV + \eta)} \right)^{n+1} \frac{\sqrt{\pi} \Gamma(n+1)}{\Gamma(n+(3/2))} \\
2F_1 \left( \frac{n+1}{2}, \frac{n+2}{2}, n+\frac{3}{2}; -\frac{kV}{s+gV+\eta} \right)^2,
\]

(9)

where \( \Gamma \) is the Gamma function, and \( 2F_1 \) is the Gauss’s hypergeometric function.

We can numerically calculate \( E_{G,n}(r, t) \) using the fast Fourier transform (FFT) algorithm over frequency and wave number for given three parameters of \( g_0, V, \) and \( Q_n^{-1} \). In Fig. 2(b), we give examples of calculated theoretical envelope Green’s functions for a double-couple source in solid curves and those for an isotropic source in broken curves. Time on the horizontal axis and energy density on the vertical axis are both normalized. Envelopes at three different receivers are shown. A prominent character of the envelope Green’s function \( E_{G,n}(r, t) \) is a long tail which follows the direct wave and decays slowly due to the scattering. Moreover, early parts of envelopes clearly exhibit a difference in energy density due to the radiation pattern. However, the difference becomes smaller as time elapses. Envelopes for the double-couple source are found to converge to those for the isotropic source after twice the direct S-wave travel time. The energy density of the higher-order modes diminishes faster than that of the lower-order modes because of multiple isotropic scattering. Therefore, only the lowest 0th mode corresponding to spherical source radiation dominates at large lapse times. From the examples, it is found that detailed information on the focal mechanism can be extracted from early part of the coda whose lapse time is smaller than twice the direct S-wave travel time.

Using this envelope Green’s function, we formulated an envelope inversion method to estimate the spatial distribution of energy radiation from an earthquake fault and site amplification factors. The method is schematically illustrated in Fig. 1. A rupture is assumed to propagate with a constant rupture velocity of \( V_r \) from the initial rupture point.
The fault plane is composed of subfaults. When a rupture front passes through the $k$th subfault, the energy $W_k$ is radiated from a double-couple source on the subfault with a time history of $f_k(t)$. The integral of $f_k(t)$ over the transit time of the rupture front is normalized as 1. The radiated energy is multiply scattered in the course of propagation through the scattering medium, and reaches the $i$th receiver at the $j$th time, and is modified by a subsurface structure in the vicinity of the receiver (Fig. 1). We further assume that seismic energies radiated from different subfaults are additive (waves are incoherent), so that the energy density for the $i$th receiver at the $j$th time is the sum of the radiated energy from the subfaults. Then, we can formulate the theoretical energy density $C_{ij}$ as

$$C_{ij} = R_i^2 \sum_k W_k F_{ijk},$$  \hspace{1cm} (10)$$

where

$$F_{ijk} = \int f_k(t') E_G(x_i - x_k, t_j - t') \, dt'.$$  \hspace{1cm} (11)$$

$F_{ijk}$ is the convolution of the envelope Green’s function and the energy radiation time history. $R_i$ is the receiver (site) amplification factor for velocity amplitude at the $i$th receiver. Under the framework of Eq. (2), this corresponds to the assumptions that $<S(t)^2> = W_k f_k(t)$, $<P(t)^2> = E_G(x, t)$, and $<R(t)^2> = R_i^2 \delta(t)$. The values of $W_k$ and $R_i$ are estimated so as to minimize the residual between observed envelopes and synthesized ones in the following least squares sense:

$$\sum_i \sum_j \left( \frac{1}{\operatorname{max}_j O_{ij}} \right)^2 |O_{ij} - C_{ij}|^2 \rightarrow \operatorname{Min}.,$$  \hspace{1cm} (12)$$

where $O_{ij}$ is the observed energy density at the $i$th receiver and the $j$th time. We normalize both the observed envelopes and the synthesized ones by the observed maximum value at each receiver to set the weight of all receivers equal. To simplify the inversion, we further assume that $f_k(t)$ is a box-car function with the same duration time of $\Delta t$ for all subfaults. Because Eq. (10) is nonlinear for the radiated energy $W_k$ and the site amplification factor $R_i$, the equation is iteratively solved by the following procedures: (i) Assuming values of $V_r$ and $\Delta t$, (ii) Setting the initial value of $R_i$ for all receivers. The value is assumed to be 2 for a reference hard rock site on the surface, and 1 for a reference hard rock site in the subsurface. (iii) Solving Eq. (10) for the radiated energy $W_k$ by the linear least squares method. (iv) Estimating the site amplification factors by fixing the radiated energy calculated in step (iii). (v) Iterating steps (iii) and (iv) until the residual between the observed envelopes and synthesized ones does not change with increasing number of iterations. We thus estimate the best-fit values of $W_k$ and $R_i$ for various sets of $V_r$ and $\Delta t$. The final result is obtained by choosing the solution having the minimum residual among them.

Finally, we mention about a few points which are necessary for practical applications of the inversion method. First, we need to make corrections of travel times and takeoff angles by using a horizontally layered structure, because theoretical envelopes are calculated for a medium with homogeneous background S-wave velocity. Second, we
need to select a reference receiver carefully to estimate absolute values of radiated seismic energy. Velocity logging data and site amplification factors estimated by other previous studies are useful for the purpose. Third, strong nonlinear site effects may affect our inversion results, because our modeling is based on the linear elastic theory. For an accurate estimation of radiated seismic energy, a reference receiver is required to never experience the nonlinear effect.

3. Data Analysis and the Results

3.1. An Example of Practical Data Analysis

We explain analysis procedures of our inversion method by taking the 2003 Miyagi-ken Oki, Japan, earthquake (Mw 7.0) as an example. This is an intraslab earthquake which took place at a depth of about 70 km in the subducted Pacific plate beneath northeastern Japan. We use strong-motion seismograms recorded at 18 stations of the K-NET and Kik-net within epicentral distance of 50 km. A reference station is set at a subsurface (depth of about 100 m) of MYGH12 station, denoted as MYGB12 in this study, because logging data show high seismic velocity at the site. Three component acceleration records are numerically integrated to velocity records, and are band-pass filtered in four octave-width frequency bands of 1–2, 2–4, 4–8, and 8–16 Hz. We square band-passed velocity records, take the sum of three components, and then smooth them by taking a moving average using a time window of 2 s. Multiplying them by a density of the crust ($2.5 \times 10^3$ kg/m$^3$), we obtain seismogram envelopes having the unit of energy density (J/m$^3$). A time window from the S-wave onset to the lapse time of 51.2 s is used for the inversion analysis. The end of the time window is set to be smaller than twice the direct S-wave travel time.

The envelope Green’s function is calculated using scattering parameters ($g_0$ and $Q_i^{-1}$) estimated at Onagawa (ONG) station (a solid square in Fig. 3) by Sakurai (1995) using envelopes of smaller events in the region. The background S-wave velocity of the scattering medium is estimated to be 4.09 km/s. Travel times and takeoff angles of S waves are corrected using a structure with four horizontal layers. For the inversion analysis, we set a fault plane with a length of 30 km and a width of 25 km dipping to the west (see Fig. 3), and divide it into 30 subfaults each of which is a $5 \times 5$ km$^2$. The geometry of the fault plane is assumed as strike = $193^\circ$, dip = $69^\circ$, and rake = $87^\circ$ with reference to a focal mechanism obtained from far-field body waves by Yagi (2003). Rupture velocity and duration time of the box-car source time function are estimated by grid search.

A contour map in Fig. 4 shows residuals between observed envelopes and synthesized ones for all the four octave-width frequency bands plotted for various rupture velocities and source duration times. A solid star marks parameters for which the minimum residual is obtained. The residuals are normalized by the minimum one. The duration time was estimated to be 1.6 s. Rupture velocity is 3.8 km/s. In Fig. 5, we show the spatial distribution of seismic energy radiation on the fault plane in a gray shade, in which a darker color corresponds to larger energy radiation. A solid star shows the initial rupture point. High-frequency seismic energy was mainly radiated from two regions on the fault. The first one is around the initial rupture point and the other is a northern deeper part of the fault. This spatial pattern looks common irrespective of frequency band analyzed. Total amount of seismic energy radiation is $8.3 \times 10^{15}$ J in 1–16 Hz. Observed envelopes
and synthesized ones (broken) for 4–8 Hz are shown in Fig. 6. Two peaks are clearly found in observed envelopes at most of the stations. Generally, the peaks are well explained by the synthesized envelopes. The first and the second peaks are attributed to the strong energy radiation from the initial rupture point and the northern deeper part of the fault, respectively.

For the estimation of errors in our inversion results, we perform the following procedure. First, we produce synthetic envelopes for the best-fit distribution of energy radiation in Fig. 5 and best-fit site amplification factors and by adding random noise to the synthetic envelopes. The random noise is assumed to obey an exponential distribution. To the amplitude of the noise, a root MS residual between an observed envelope and a synthesized envelope from the best-fit solutions in Fig. 5 is assigned at each station. The amplitude of the noise is up to 15% of the maximum amplitude of the signal. Repeating the envelope inversion by changing random noise 100 times, we estimate the spatial distribution of energy radiation. In Fig. 7, we show standard deviation of
estimated seismic energy normalized by the true solution in Fig. 5, that is, the coefficient of variation (CV). Errors are considered to be small in the parts where CV is small. Two parts of strong high-frequency energy radiation shown in Fig. 5 are found to be located in the region shaded by black, which confirms that estimation errors in seismic energy are less than 20% for the two parts.

From a waveform inversion analysis of both teleseismic and near-field seismograms in a frequency range between 0.05 Hz and 0.5 Hz, Yagi (2003) estimated the spatial distribution of slip on the fault plane. A contour of the slip is shown in Fig. 8. The maximum slip amount reaches about 1.7 m. From the comparison between the slip distribution and high-frequency seismic energy radiation (shaded map in Fig. 8), both the high-frequency and the low-frequency waves are radiated around the initial rupture point. But the other region of high-frequency radiation in the northwestern part does not overlap an asperity and rather corresponds to the edge of the asperity. Therefore, the spatial relationship between the location of high-frequency radiation and that of low-frequency radiation is not simple for this event.

4. Compilation of the Results

We have applied the envelope inversion method of Nakahara et al. (1998) to nine earthquakes around Japan with moment magnitude (Mw) of from 5.9 to 8.3. Among the earthquakes, three are interplate earthquakes, one is an intraslab earthquake, and five are
inland earthquakes. Focal mechanisms differ among the earthquakes. Although the number of nine cases is small, we compile the results in Table 2, and make one of the first trials to extract statistical characteristics of high-frequency seismic energy radiation from moderate-to-large earthquakes. We put our focus on the following three subjects: (1) Frequency dependence of high-frequency radiated energy. (2) A scaling relationship between high-frequency radiated energy and earthquake magnitude. (3) A spatial relationship between locations of asperities (areas of large fault slip) and locations of high-frequency energy radiation. The second subject can be studied only by our envelope inversion analysis, because our method is capable of dealing with absolute values of seismic energy.

4.1. Frequency Dependence of High-Frequency Seismic Energy

First, we examine the theoretical frequency dependence of high-frequency seismic energy. If the source spectrum obeys the omega-squared model (e.g., Aki, 1967;
4–8 Hz, \( V_r = 3.8 \text{ km/s}, \text{ Dur.} = 1.60 \text{ s}, \text{ Res.} = 21.33 \)

**Fig. 6.** Comparison between observed envelopes (solid curve) and synthesized ones (broken curve) for the 4–8 Hz band. Synthesized envelopes explain observed ones well.
Brune, 1970), the acceleration source spectrum \( a(\omega) \) becomes flat at frequencies higher than the corner frequency, and the amplitude level, often denoted as \( A \), can be expressed as

\[
a(\omega) \equiv A \propto \sigma L,
\]  

(13)

where \( \omega \) is the angular frequency, \( \sigma \) is a stress parameter, and \( L \) is a characteristic length scale of a fault. If the stress parameter \( \sigma \) is constant, \( A \) is proportional to \( L \). A velocity source spectrum \( v(\omega) \) in the frequency range is

\[
v(\omega) = \frac{A}{\omega},
\]  

(14)

S-wave energy spectrum in an octave-width frequency band \( E_{\text{HF}}(\omega) \), which is directly obtained from our envelope inversion method, is expressed as

\[
E_{\text{HF}}(\omega) = \frac{1}{10\pi \rho \beta^5} \int_{2\omega}^{20\omega} \left| v(\omega') \right|^2 d\omega' = \frac{1}{20\pi \rho \beta^5} \frac{A^2}{\omega},
\]  

(15)

where \( \rho \) is the density and \( \beta \) is the S-wave velocity. Therefore, the seismic energy is expected to decrease with frequency with a power of \(-1\).
In Fig. 9, we plot $E_{HF}$ versus frequency relations for nine earthquakes shown in Table 2. Seismic energy in an octave-width band is found to decrease with increasing frequency. Here, we fit regression lines to the data to estimate the power of the decay of seismic energy with frequency. The results are tabulated in Table 3. The mean value of the power for all the events is estimated to be $-0.99$ which is close to the theoretical expectation of $-1$, though it shows variation from $-0.38$ for the Sanriku event to $-1.85$ for the Kobe event. The median value is $-0.70$. To have a more closer look at the frequency dependence, we find a slight increase of energy in 1–8 Hz band for the Sanriku event. We also note that the energy in the 8–16Hz band falls off rapidly for the Sanriku event and the Kobe event. Although someone might think that this is caused by $f_{\text{max}}$, the two datasets are probably contaminated by instrumental response which we could not correct. For events other than Sanriku and Kobe, we can not detect a rapid fall off for higher frequency which might be associated with $f_{\text{max}}$.
Here, we discuss a scaling relationship between high-frequency seismic energy and earthquake magnitude. As shown in the previous subsection, our observation of high-frequency seismic energy in an octave-width frequency band for individual events shows variation from the theoretical expectation based on Eq. (15). However, it may be acceptable to use the Eq. (15) as a reference since it fits the average of the observed data. For a fixed frequency band, we obtain the following relation by taking the logarithm of both sides of Eq. (15)

$$\log E_{HF}(\nu) \propto 2 \log A \sim \log L^2 \sim M. \quad (16)$$

The final proportionality comes from a well-known empirical relationship between earthquake magnitude and fault area (e.g., Kanamori and Anderson, 1975). Therefore, the logarithm of high-frequency seismic energy in an octave-width band is predicted to be proportional to earthquake magnitude. Or, high-frequency seismic energy is proportional to fault area. This is an important relation which only holds for high-frequency seismic energy. It should be noted that the coefficient of proportionality is 1, which is different from 1.5 in the Gutenberg–Richter’s relation (Gutenberg and Richter, 1956) for whole-band seismic energy. We also find that the high-frequency seismic energy becomes half when a frequency is doubled.
Frequency dependence of high-frequency seismic energy for nine earthquakes. Solid circles are data and solid lines are regression lines. High-frequency seismic energy in an octave-width frequency band decreases with increasing frequency.

Table 3. Power of the decay of high-frequency seismic energy with frequencies

<table>
<thead>
<tr>
<th>Event (Mw)</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>1994 Off-Sanriku, JAPAN (7.6)</td>
<td>$-0.38 \pm 0.41$</td>
</tr>
<tr>
<td>1995 Kobe, JAPAN (6.9)</td>
<td>$-1.85 \pm 0.58$</td>
</tr>
<tr>
<td>1998 Northern Iwate, JAPAN (5.9)</td>
<td>$-0.91 \pm 0.26$</td>
</tr>
<tr>
<td>1999 Chi-Chi, Taiwan (7.6)</td>
<td>$-1.83 \pm 0.17$</td>
</tr>
<tr>
<td>2000 Western Tottori, JAPAN (6.7)</td>
<td>$-1.08 \pm 0.15$</td>
</tr>
<tr>
<td>2003 Off-Miyagi, JAPAN (7.0)</td>
<td>$-0.66 \pm 0.13$</td>
</tr>
<tr>
<td>2003 Off-Tokachi, JAPAN (8.3)</td>
<td>$-0.79 \pm 0.15$</td>
</tr>
<tr>
<td>Largest aftershock of the 2003 Off-Tokachi, JAPAN (7.3)</td>
<td>$-0.76 \pm 0.28$</td>
</tr>
<tr>
<td>2004 Niigata Chuetsu, JAPAN (6.6)</td>
<td>$-0.70 \pm 0.14$</td>
</tr>
</tbody>
</table>
Then, we compile the results for nine earthquakes shown in Table 2. Here, we divide the data into two categories: one is inland earthquakes and the other is offshore earthquakes. For offshore earthquakes, there is no discrimination between interplate earthquakes and intraplate ones. Observed high-frequency seismic energy in 1–2, 2–4, 4–8, and 8–16 Hz, respectively, is shown against Mw in Fig. 10. Observed data are shown by open symbols. For the calculation of theoretically expected values from the Eq. (15), we assume that $\rho = 2.5 \times 10^3 \text{ [kg/m}^3\text{]}$ and $\beta = 3.5 \text{ [km/s]}$. In addition, we adopt a relation $A = 5.3 \times 10^{12} M_0^{1/3} \text{ [Nm/s}^2\text{]}$ in which $M_0$ is measured in [Nm], corresponding to Brune’s stress drop of 9.7 [MPa] (97 [bars]). The value was obtained from measurements for 12 inland earthquakes by Dan et al. (2001). Theoretically expected values thus calculated are shown by solid, long-broken, short-broken, and dotted lines for 1–2, 2–4, 4–8, and 8–16 Hz, respectively. Our observation matches with the expectation for inland earthquakes as shown in Fig. 10(a). This suggests that our estimates for high-frequency seismic energy are consistent with independent estimates by Dan et al. (2001) which used a different method. However, we cannot explain levels of high-frequency seismic energy for offshore earthquakes in Fig. 10(b) by using $A$ of Dan et al. (2001). Our observation seems larger than the expectation by about 10 times. Because the energy is proportional to $A^2$ as shown in Eq. (15), we multiply the $A$ value by 3.16 (square root of 10), corresponding to Brune’s stress drop is about 54.6 [MPa] (546 [bars]), and compare again with our observation for offshore earthquakes in Fig. 10(c). The new expected values can roughly explain our observation for the offshore earthquakes. This is not a strict fit to the data but a rough estimate. However, this implies that the offshore earthquakes in northeastern Japan radiate about 10 times more high-frequency seismic energy than inland earthquakes in Japan and Taiwan with the same magnitude. This tendency was also reported for the same region by Satoh (2004) and for regions to the south by Takemura et al. (1989) and Kato et al. (1998), though it seems to contradict an empirical rule that the static stress drop is higher for intraplate events than for interplate events (e.g., Kanamori and Anderson, 1975). Kato et al. (1998) referred to the depth dependence of the stress drop as a possible cause.

Two points have been clarified from our studies. First, the high-frequency seismic energy is proportional to fault area. This relation is a manifestation that the high-frequency seismic waves are incoherent. Because this point was first pointed out by Koyama and Zheng (1985), our result is a confirmation of their results. Second, there exists a regional difference in the excitation level of high-frequency seismic energy. Although the reason is not clear, this result is practically important for quantitative prediction of strong ground motion.

4.3. Spatial Relationship Between Asperities and High-Frequency Sources

For broadband simulations of strong ground motion due to an earthquake fault, it is necessary to specify regions of high-frequency radiation on the fault as well as those of low-frequency wave radiation (asperities). So, a spatial relationship between these two kinds of regions is of particular interests in strong-motion seismology.

Here, we classify the relation into 3 cases: (1) Complementary, (2) Matching, and (3) Otherwise. The complementary case means that locations of high-frequency radiation are at peripheries of asperities. The matching case means the both locations are the same. The otherwise case includes any others but the two. For example, if there are several
Fig. 10. Scaling relation of high-frequency seismic energy radiation in four octave frequency bands with moment magnitude. Open circles, open triangles, open squares, and open inverted triangles are observed energy for 1–2, 2–4, 4–8, and 8–16 Hz, respectively. Solid, long-broken, short-broken, and dotted straight lines with a proportionality factor of 1 show theoretically expected values for 1–2, 2–4, 4–8, and 8–16 Hz, respectively. Figure (a) shows the results for 5 inland earthquakes with the expected values from the $A$ value of Dan et al. (2001). Figure (b) is for offshore earthquakes composed of 3 interplate earthquakes and 1 intraslab earthquake with the expected values from the $A$ value of Dan et al. (2001). Figure (c) is for the offshore earthquakes composed of 3 interplate earthquakes and 1 intraslab earthquake with the expected values from 3.16 (square root of 10) times the $A$ value of Dan et al. (2001).
asperities and high-frequency sources on a fault plane, some are complementary but
some are matching. Such a case is identified as the otherwise case. In terms of this
classification, four events are identified as complementary, none is matching, and four
are otherwise. Unfortunately, the comparison is impossible for one event because no fault
slip models have been estimated for lower frequencies.

According to envelope inversion studies by other groups (shown in Table 4), the
complementary relation is reported for the 1989 Loma Prieta earthquake (Ms 7.1) (Zeng
et al., 1993) and for the 1993 Kushiro-Oki, Japan, earthquake (Mw 7.6) (Kakehi and
Irikura, 1996). The otherwise relation is reported for the 1993 Hokkaido-Nansei-Oki,
Japan, earthquake (Mw 7.7) (Kakehi and Irikura, 1997), for the 1994 Northridge earth-
quake (Ms 6.7) (Hartzell et al., 1996) and the 1995 Kobe, Japan, earthquake (Mw 6.9)
(Kakehi et al., 1996).

On the contrary, from the simultaneous fitting of displacement waveforms and accel-
eration envelopes in a 0.2–10 Hz band for 12 crustal earthquakes (Mw 4.8 – 6.0) in Japan,
Miyake et al. (2003) estimated source areas, which they call the strong-motion genera-
tion areas, on each fault plane. They found that the strong-motion generation areas
coincide with those of asperities.

The classification conducted in this subsection is still qualitative. We hope to intro-
duce more quantitative measures to characterize the relationship between locations of
low-frequency radiation and those of high-frequency radiation in the near future. In terms
of this viewpoint, Gusev et al. (2006) conducted a quantitative analysis. They estimated
slip rate time functions in lower frequencies and seismic luminosity (source) time
functions in high frequencies (0.5–2.5 Hz) at the same time for 23 intermediate-depth
earthquakes with magnitude of 6.8 and larger. Calculating correlation coefficients
between the two kinds of time functions after corrections of propagation effects, they
estimated the mean correlation coefficient of 0.52, and concluded that this mean value
indicates the genuine difference in distributions of low-frequency radiation and high-
frequency radiation.

Finally, we discuss this spatial relation based on kinematic heterogeneous fault-
rupture models (e.g., Papageorgiou and Aki, 1983; Koyama, 1985). In the models,

<table>
<thead>
<tr>
<th>Event (Mw)</th>
<th>Source location</th>
<th>Focal mechanism type</th>
<th>HF Location and LF location</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1989 Loma Prieta (6.9)</td>
<td>Inland</td>
<td>Reverse</td>
<td>Complementary</td>
<td>Zeng et al. (1993)</td>
</tr>
<tr>
<td>1993 Kushiro-Oki, JAPAN (7.6)</td>
<td>Intraslab</td>
<td>Down-dip extension</td>
<td>Complementary</td>
<td>Kakehi and Irikura (1996)</td>
</tr>
<tr>
<td>1993 Hokkaido-Nansei-Oki, JAPAN (7.7)</td>
<td>Plate boundary</td>
<td>Thrust</td>
<td>Otherwise</td>
<td>Kakehi and Irikura (1997)</td>
</tr>
<tr>
<td>1994 Northridge, California (6.6)</td>
<td>Inland</td>
<td>Reverse</td>
<td>Otherwise</td>
<td>Hartzell et al., (1996)</td>
</tr>
<tr>
<td>1995 Kobe, JAPAN (6.9)</td>
<td>Inland</td>
<td>Right-lateral strike slip</td>
<td>Otherwise</td>
<td>Kakehi et al. (1996)</td>
</tr>
</tbody>
</table>

HF, high frequency; LF, low frequency.
many small-scale asperities or patches are assumed to be randomly distributed on the fault. Generation of high-frequency seismic waves is due to the rupture of the small-scale asperities. On the contrary, low-frequency waves are radiated by the coherent rupture of the entire fault. Relative contribution of the coherent rupture and the incoherent rupture may be a key. If the coherent rupture is dominant, we expect that the rupture propagates smoothly and stops at the edge of the fault. For example, Madariaga (1976) investigated the dynamic rupture of a circular crack. He showed that slip (moment release) is large at the center of the fault and high-frequency seismic waves are strongly radiated from edges of the fault. This is an example of a complementary relation. On the contrary, if the contribution of the incoherent rupture becomes large, we expect that high-frequency waves are strongly radiated from throughout the fault plane due to the rupture of small-scale asperities, whereas slip is large at the center of the fault due to the coherent rupture. This is an explanation for the matching case. Based on the consideration, the relationship between the locations of high- and low-frequency radiation may be understood by a relative weight of the coherent rupture and the incoherent rupture, which may be controlled by the heterogeneity of stress and/or strength on the earthquake fault: Complementary for homogeneous faults and matching and/or otherwise for heterogeneous faults. But the consideration here is under a frame of kinematic rupture models. Dynamic rupture simulations should be conducted for asperities with various degrees of heterogeneities in stress and/or strength for more quantitative considerations. Kato (2007) may be the first step forward to this direction.

5. Conclusions

We have made a brief review of envelope inversion studies for high-frequency seismic wave radiation from moderate-to-large earthquakes. Several methods have been proposed since the 1990s. An assumption on the incoherency of high-frequency seismic waves facilitates direct convolution of each envelope for source, path, and site effects. Thanks to the methods, it became possible to image earthquake source process at high frequencies and to compare the results to those from lower frequencies. On the basis of results for 9 earthquakes so far analyzed by us, we have clarified the following two characteristics in high-frequency seismic energy radiation. First, logarithm of the high-frequency seismic energy is proportional to the moment magnitude with a coefficient of proportionality of 1 as is theoretically expected. Moreover, a regional difference in the high-frequency seismic energy radiation has been detected: Earthquakes in offshore regions in northeastern Japan are found to be more energetic by about an order of magnitude than inland earthquakes in Japan and Taiwan. Second, spatial relationships between the locations of asperities and the locations of high-frequency radiation have been summarized. Among 9 earthquakes analyzed by us, 4 are complementary, none is matching, 4 are otherwise, and 1 is indeterminate. According to analyses of 5 earthquakes by other groups, 2 are reported to be complementary, none is matching, 3 are otherwise. In total, 6 are complementary, none is matching, 7 are otherwise, and 1 is indeterminate among 14 earthquakes. Reasons for the variation are not yet known. However, heterogeneities in the distribution of stress, strength, and material properties on and around earthquake faults may control the variation. The two characteristics found for high-frequency seismic energy radiation will give important information for the study of high-frequency earthquake source processes.
And they may also contribute to improving the accuracy of predicting strong ground motion, because the locations of high-frequency radiation on an earthquake fault greatly affect ground motion at nearby stations.

Acknowledgments

Most of the results were obtained through collaborations with H. Sato, M. Ohtake, T. Nishimura, and R. Watanabe. We thank the National Research Institute for Earth Science and Disaster Prevention, Japan for providing us with strong-motion data recorded by the K-NET and the Kik-net. A figure of the slip distribution for the 2003 Miyagi-oki earthquake was provided by Y. Yagi of Tsukuba University. We greatly appreciate thoughtful comments from an associate editor, M. Fehler, and two reviewers, A. Gusev and Y. Zeng.

References


Nakahara, H., Watanabe, R., Sato, H., Ohtake, M. (2006). Spatial distribution of high-frequency seismic energy radiation on the fault plane of the 1999 Chi-Chi, Taiwan, earthquake (Mw 7.6) as revealed from an envelope inversion analysis, Submitted to PAGEOPH.


