Testing Equipartition for S-Wave Coda Using Borehole Records of Local Earthquakes

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Abstract  Equipartition is a state in which energy of all possible wave modes becomes equal due to multiple scattering among the modes. This is important to identify the scattering regime and constitutes the basic principle underlying seismic interferometry. In this study, we calculate partitioning of total kinetic energy into the horizontal and vertical components, similar to the horizontal-to-vertical ratio, for S-wave coda of 60 local earthquake records at three borehole stations in Japan. S-wave coda in lapse times of 40–80 s and in the 2–16 Hz frequency band is used. At the surface receivers of two rock sites, the horizontal components are dominant and their contribution gradually increases with frequency. At a soft site, the energy partitioning shows stronger variations with frequency, reflecting the presence of low-velocity layers. Subsurface receivers at about 100 m depths show larger contribution of the horizontal components, irrespective of site conditions. We quantitatively explain the observations with random wave fields at equipartition in horizontally layered structures derived from well logging. Through the modeling, we test three formulations of equipartition: a mixture of (1) only body waves, (2) only surface waves, and (3) both body and surface waves. Subsurface receivers are critically important to distinguish among the three formulations. For S-wave coda at frequencies lower than about 5 Hz, equipartition holds among both body and surface waves. For higher frequencies, equipartition holds predominantly among body waves. Equipartition of coda waves enables a simple modeling of the horizontal-to-vertical ratio, which contrasts with ambient noise.

Introduction

Equipartition is a state in which energy of all possible wave modes becomes equal due to the presence of heterogeneities in the medium. A wave field is considered to be fully randomized in phase space in the equipartitioned state (e.g., Weaver, 1982). Practically, equipartition is an important ingredient of the noise correlation technique (e.g., Shapiro and Campillo, 2004) and of seismic interferometry (e.g., Wapenaar and Fokkema, 2006). By using modal expansions of scalar wave fields, Lobkis and Weaver (2001) proved that equipartition (i.e., a diffuse wave field) is necessary for seismic interferometry. In terms of propagating scalar waves, the isotropic incidence of random waves expected at equipartition is necessary for the complete retrieval of the Green’s function (e.g., Nakahara, 2006). Concerning propagating vector waves, Sánchez-Sesma and Campillo (2006) proved that the ratio of S-wave energy to P-wave energy (S-to-P energy ratio) expected at equipartition is necessary to retrieve the elastic Green’s function in a strict sense, in addition to an isotropic distribution of incidence angles. Therefore, equipartition constitutes a basic principle underlying seismic interferometry.

Equipartition is also important to elucidate whether single scattering or multiple scattering is predominant in the seismic coda, because equipartition is realized as a result of multiple conversions among different wave modes, caused by the scattering off lateral heterogeneities in the medium. The S-to-P energy ratio is known to be stable at equipartition (e.g., Weaver, 1982). Conversely, stability of the ratio is used as an indicator of the multiple scattering regime (Shapiro et al., 2000). The temporal evolution of the S-to-P energy ratio in scattering media can be modeled with elastic radiative transfer theory (Ryzhik et al., 1996; Margerin et al., 2000; Przybilla and Korn, 2008). The theory shows that the S-to-P energy ratio at equipartition is controlled by the P and S velocity structures independent of initial conditions, and the speed of transition toward equilibrium is governed by conversion scattering coefficients, which depend on small-scale heterogeneities in the velocity structures.

In addition to the S-to-P energy ratio, equilibration of the horizontal-to-vertical kinetic energy ratio (hereafter H/V) for S-wave coda can be also useful to check equipartition (Hennino et al., 2001). Independently, H/V has been studied for the purpose of characterizing site conditions using ambient noises (e.g., Nogoshi and Igarashi, 1971; Nakamura, 1989) and direct S waves (e.g., Lermo and Chávez-García,
In contrast with previous studies, we take advantage of the stability of H/V measured on S-wave coda. The stability results from the equilibration process caused by multiple scattering and leads to more reliable characterizations of sites. The value of H/V at equilibrium is known to be affected by the free surface (Weaver, 1985) and by layered velocity structures (Margerin, 2009). Margerin et al. (2009) studied H/V for S-wave coda using records from surface receivers and modeled the observations with the aid of equipartition theory in layered media. Hoshiba et al. (2003) studied H/V for S-wave coda using borehole records in Japan and pointed out that energy is almost equally partitioned into the three directions of motion at subsurface receivers. However, no quantitative modeling was performed. In this work, we study the relative contributions of the horizontal and vertical components of motion to the total kinetic energy (akin to H/V) from borehole records of local earthquakes in Japan. We conduct a test of equipartition theory for S-wave coda by quantitatively modeling the observations. We point out that H/V is much more sensitive to the constituents of the wave field at borehole receivers than at the free surface.

Data Analysis

Data

The three stations used in this study (IWTH02, IWTH13, and IWTH17) are located in northeastern Japan and are depicted by solid triangles in Figure 1. They belong to the KiK-net, a Japanese nationwide strong-motion network managed by the National Research Institute for Earth Science and Disaster Prevention. At each station, two three-component strong-motion seismometers are deployed at the top and bottom of a borehole with a depth of around 100 m. The seismometers installed at the bottom of each borehole are named IWTH02, IWTH13, and IWTH17 and are located at depths of 102, 117, and 103 m, respectively. Frequency characteristics of the seismometers are flat in amplitude from DC to 30 Hz (see the Data and Resources section). Acceleration records are sampled at a rate of 200 Hz. The epicenters of the 60 events used in our analysis are shown in Figure 1 by different symbols with different size, depending on their magnitude and depth. The magnitudes range from 3.3 to 7.1, the depths vary between 11 and 122 km, and epicentral distance is smaller than 200 km.

The three stations are situated in the Katakami mountain range, which is mainly composed of sedimentary and granitic rocks from Paleozoic or Mesozoic times. Seismic velocity structures derived from well logging are shown in Figure 2. S-wave velocity is higher than 1200 m/s at depths larger than a few meters at stations IWTH13 and IWTH17, which will be termed “rock sites” in this study. On the other hand, station IWTH02 shows significantly lower S-wave velocity at shallow depths and will be termed a “soft site.”

Analysis

After baseline correction, we integrate original acceleration records to obtain velocity records. This processing allows us to estimate the kinetic energy of motion by squaring the traces. Sample velocity records at station IWTH17 for an M 5.4 event are shown in Figure 3. In this figure, amplitude is normalized by the maximum for each component. We use S wave coda in a lapse-time range from 40 to 80 s. The mean free time of an S wave (i.e., the average time between two scattering events) is of the order of 30–50 s in the region under study (Hoshiba, 1993). Sliding a time window of 5.12 s by 5 s within this lapse-time range, we...
Two horizontal components (HR) as follows: lines are used. (UD, up–down; NS, north–south; EW, east–west.)

Calculate power spectra for the three components by fast Fourier transform. Next, we calculate the energy partition ratio (PTN) between the vertical component (UD), and the two horizontal components (HR) as follows:

$$\text{PTN}_{\text{UD}}(\omega; t) \equiv \frac{E_{\text{UD}}(\omega; t)}{E_{\text{UD}}(\omega; t) + E_{\text{NS}}(\omega; t) + E_{\text{EW}}(\omega; t)}$$  \hspace{1cm} (1)

and

$$\text{PTN}_{\text{HR}}(\omega; t) \equiv \frac{E_{\text{NS}}(\omega; t) + E_{\text{EW}}(\omega; t)}{E_{\text{UD}}(\omega; t) + E_{\text{NS}}(\omega; t) + E_{\text{EW}}(\omega; t)}$$  \hspace{1cm} (2)

where $E_i(\omega; t)$ denotes the power spectrum for the $i$-th component velocity seismogram at angular frequency $\omega$ for a time window centered at lapse time $t$. The power spectrum is smoothed by operating the Hanning filter 10 times, which is equivalent to applying a Parzen window. Combining all events and time windows, we calculate the mean and standard deviation of the PTN. Unlike other studies based on H/V or its reciprocal, we favor these quantities because the PTNs defined in equations (1) and (2) are always bounded between 0 and 1. This property is advantageous for a quantitative fit, although the ratios PTN$_{\text{UD}}$ and PTN$_{\text{HR}}$ are mutually dependent through the constraint

$$\text{PTN}_{\text{UD}}(\omega; t) + \text{PTN}_{\text{HR}}(\omega; t) = 1.$$  \hspace{1cm} (3)

Results

Figure 4 shows observed PTNs for all events and time windows at the surface receivers. The top and bottom panels show the PTNs for the two HR components and the UD component, respectively. At the two rock sites IWTH13 and IWTH17, PTNs for the HR components are 70–80% below 5 Hz and become gradually larger with increasing frequency. In other words, the PTN for the UD component is about 20–30% below 5 Hz and gradually decreases as frequency increases. As a reference, the expected PTNs at the free surface of a homogeneous half-space for isotropic illumination of body waves are shown. PTN is 0.82 for the HR components and 0.18 for the UD component (Weaver 1985, Hennino et al., 2001). For frequencies lower than about 5 Hz, the observed PTN appears to match these values. A more quantitative

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{An example of velocity (Vel.) records at station IWTH17 for an $M$ 5.4 event with an epicentral distance of about 110 km. The record of each component is normalized by its maximum amplitude. A direct $P$ wave appears at a lapse time of about 20 s. Arrival of a direct $S$ wave is shown by dashed lines at a lapse time of about 30 s. Dashed lines occurring again at about 60 s mean twice the direct $S$-wave travel time. $S$-wave coda in lapse times between 40 and 80 s sandwiched between two vertical solid lines are used. (UD, up–down; NS, north–south; EW, east–west.)}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{Observed energy partitioning ratio of (top) two horizontal (HR) components and (bottom) vertical (UD) component at three surface receivers. Results at IWTH13, IWTH17, and IWTH02 are shown from left to right. Open circles and error bars stand for mean and one standard deviation for all events and all time windows. Expected values on the homogeneous free surface for isotropic illumination of body waves are shown by solid horizontal lines: 0.82 for the horizontal components and 0.18 for the vertical component.}
\end{figure}
comparison is made in the next section (Testing the Equipartition Hypothesis). At higher frequencies, PTNs show larger contributions of the HR components. On the other hand, at station IWTH02 (the soft site), the PTN exhibits much larger variations with frequencies. The PTN of the UD component is small even at frequencies lower than 5 Hz and shows a peak around 11 Hz. From the P-wave velocity structure shown in Figure 6b, we infer that the predominant frequency of P waves that are normally incident and reverberated in the shallowest low-velocity layer with a thickness of 5 m is about 15 Hz. As further substantiated in the Discussion section, the observed peak can be considered as a consequence of P-wave reverberation in the low-velocity layer.

Figure 5 shows the observed PTNs for all events and time windows at the subsurface receivers. At all receivers, the frequency dependence seems similar irrespective of site conditions. At frequencies lower than about 5 Hz, the observed PTN is close to two-thirds for the HR components, which agrees with simple equipartition in a homogenous full-space. This agreement is coincidental as shown in the next section, Testing the Equipartition Hypothesis. At higher frequencies, PTNs show larger contributions of the HR components. On the other hand, at frequencies lower than 5 Hz and shows a peak around 11 Hz. From the P-wave velocity structure shown in Figure 6b, we infer that the predominant frequency of P waves that are normally incident and reverberated in the shallowest low-velocity layer with a thickness of 5 m is about 15 Hz. As further substantiated in the Discussion section, the observed peak can be considered as a consequence of P-wave reverberation in the low-velocity layer.

Testing the Equipartition Hypothesis

In this section, we try to explain the observed PTN by modeling random elastic wave fields at equipartition in a layered medium, as shown in Figure 6b. Let us briefly discuss the underlying physical assumptions. As shown in Figure 6a, we analyze coda waves from many local earthquakes (shown by solid stars) in a heterogeneous medium where scatterers (shown by open circles) are embedded in a horizontally layered structure. Conversion scattering among body waves and surface waves caused by the scatterers facilitates the transition to equipartition. In addition, the use of different earthquakes should also help satisfy the equipartition hypothesis. If so, we will be able to explain the observed PTN averaged over many earthquakes and time windows.

Figure 5. Observed energy partitioning ratios of (top) two horizontal (HR) components and (bottom) vertical component at three subsurface receivers. Results at IWTH13, IWTH17, and IWTH02 are shown from left to right. Open circles and error bars stand for mean and one standard deviation for all events and all time windows. The expected value for simple equipartition in a homogeneous full-space is shown by solid horizontal lines: 0.67 for the horizontal components and 0.33 for the vertical component.
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where $x$ and $y$ are horizontal coordinates, and $z$ is positive downward. Wavenumber is expressed as $k$, eigenfunction is denoted by $\psi$, and the amplitude of each mode is a complex random variable denoted by $A$. Superscripts $R$ and $L$ stand for Rayleigh and Love waves, respectively. The superscripts $P$ and $S$ indicate that the mode is composed of a propagating $P$- or $S$-wave incident from the underlying half-space, together with the $P$ and $S$ waves reflected from the stack of layers. Note that the reflected $P$ wave may be evanescent in the lower half-space. The body-wave modes are not of integrable square and verify the normalization of the continuum (see Margerin, 2009, for details). On the right side of equation (4), the wave field is represented as a sum of Rayleigh waves (first term), Love waves (second term), and body-wave modes of $P$ type (third term) and $S$ type (fourth term). The index $n$ serves to label the surface-wave mode, with $n = 0$ for the fundamental mode, $n = 1$ for the first higher-mode, and so on. The total number of modes in the sum depends on the velocity structure and the target frequency.

At equipartition, the following three relations are realized:

$$\langle A_n^{R,L} A_{n'}^{P,S} \rangle = 0,$$

where $\langle \rangle$ stands for an ensemble average. In practice the average is performed over different sources and time windows. Relation (5) means that surface-wave modes are statistically independent from body-wave modes.

$$\langle A_n^{R,L} (p_x, p_y, p_z) A_{n'}^{R,L} (p'_x, p'_y, p'_z)^* \rangle = \sigma^2 \delta^{R,L} \delta_{nn} \delta (p_x - p'_x) \delta (p_y - p'_y),$$

where $\delta_{nn}$ is Kronecker’s delta and $\delta^{R,L}$ equals 1 for surface waves of the same type and 0 otherwise. According to this relation, surface waves with different wavenumbers and/or different polarizations are mutually independent. Each mode of surface waves is generated with equal power $\sigma^2$.

$$\langle A_n^{P,S} (p_x, p_y, p_z) A_{n'}^{P,S} (p'_x, p'_y, p'_z)^* \rangle = \sigma^2 \delta^{P,S} \delta (p_x - p'_x) \delta (p_y - p'_y) \delta (p_z - p'_z),$$

where $\delta^{P,S}$ equals 1 for the same types of body waves and 0 otherwise. Equation (7) implies that body-wave modes with different wavenumbers and/or different polarizations are mutually independent. Again each mode is generated with equal power $\sigma^2$.

Regarding the constituents of the wave fields, we test three different assumptions based on equipartition: (1) equipartition among only body waves, (2) equipartition among only surface waves, and (3) equipartition among both surface and body waves. If we are able to explain the measured PTN using these assumptions, equipartition is a necessary condition for our observations. Clearly, equipartition in a strict sense corresponds to assumption (3). However, when considering application to data, it is important to take into account the fact that the coda wave field may be imperfectly equipartitioned. There are two main reasons for this: (1) Although equipartition has a universal character, this dynamic process takes a certain time to set in, which depends on the local characteristics of the scatterers and on the composition of the incident wave field (Margarin et al., 2000). In the case of remotely detected earthquakes, we may expect the incident coda wave field to be imperfectly diffuse. For instance, it may lack surface waves at high frequency because they propagate in shallow low-$Q$ material. As shown for instance at Pinyon Flats by Vernon et al. (1998), the surface waves present in the coda are generated locally and it may take some time before all the surface-wave modes reach complete equipartition. (2) An additional problem which in our opinion is not completely solved theoretically, is the fact that equipartition may break down in absorbing media. Some preliminary results on this issue have been obtained by Margerin et al. (2001), who showed that, if a wave mode is strongly absorbed, it may be absent from the coda wave field. Again this implies that it may be relevant to consider different assumptions on the composition of coda waves. In this work, we consider some end-member cases for illustrative purposes.

Information on the depth dependence of mass density is required for the calculation but is not provided by the logging data. Therefore we adopt the following empirical relation between density $\rho$ (in kilograms per cubic meter) and $P$-wave velocity $V_p$ (in meters per second) (Gardner et al., 1974) as

$$\rho = 310V_p^{0.25}.$$

In Figures 7 and 8, we compare the observed PTN with the synthesized one for the three equipartition assumptions at

(a) Free Surface

(b) Free Surface
surface and subsurface receivers, respectively. The Hanning filter is also applied 10 times to the synthetics. In each panel, dotted curves show synthetics for assumption (1), dashed curves for assumption (2), and solid curves for assumption (3). At stations IWTH13 and IWTH17, in addition to body waves, only the fundamental mode Rayleigh and Love waves exist for frequencies lower than about 10 Hz. For station IWTH02, the first three harmonics of Rayleigh waves and the first two harmonics of Love waves compose the surface waves in the same frequency range. Figure 7 shows that the synthetics for all three assumptions generally explain the PTN measured at surface receivers. At frequencies lower than about 5 Hz, the fit is acceptable within the one-standard deviation confidence interval. At higher frequencies, the fit is deteriorated. There is room for a better fit by modifying the velocity structures at shallower depths, especially at IWTH02. We briefly discuss this point later. Overall, it may come as a surprise to obtain such a good fit without any adjustment of the velocity structures derived from well logging. We also observe that it is not easy to say something about the constituents of coda wave fields from surface observations only, as the difference between the three different assumptions is generally small. On the other hand, Figure 8 shows a clear difference between the synthetics calculated at the borehole receivers for the three assumptions. For all three subsurface receivers, the solid curves corresponding to assumption (3) exhibit the best fit at frequencies lower than about 5 Hz. For higher frequencies, the dotted curves corresponding to assumption (1) match the observations best. The dashed curves corresponding to assumption (2) are generally away from the observation. Therefore, coda waves are likely to be composed of equipartitioned body and surfaces waves at frequencies lower than about 5 Hz. The contribution of surface waves becomes smaller with increasing frequency. So far, the reason is not clear. High attenuation at shallower depths might be a cause for the smaller contribution of surface waves at higher frequencies. Yet another possibility is the decreasing efficiency of the excitation mechanism of surface waves in the high-frequency coda. The two effects may operate simultaneously.

Discussion

Toward Inversion of PTN for the Subsurface Velocity Structure

Though inversion of PTN for the subsurface layered velocity structure is beyond the scope of this study, we briefly mention this possibility. As shown in Figure 7, the peak around 11 Hz seen on the PTNUD curve at station IWTH02 is not well explained because the peak shows up at a frequency of about 15 Hz in the synthetics. In order to better fit the peak frequency, we decrease the $P$-wave and $S$-wave velocity in the shallowest layer from 300 m/s and 150 m/s to 230 m/s and 115 m/s, respectively. The layer thickness (5 m) is kept constant. Because of this change, the resonance frequency for a normally incident $P$ wave decreases from 15 to 11 Hz. The comparison between the synthetics for the modified structure and the observations is shown in Figure 9. The fit of the high-frequency peak is clearly improved. It is interesting to note that the resonance frequency of the shallow layer also coincides roughly with an inversion of the Rayleigh wave ellipticity. The two effects may contribute to the observations, but the overall shape of the peak is much better described by the body-wave assumption. In Figure 9, we also show the fit at the subsurface receiver IWTB02. We do not observe any significant improvement compared to Figure 8. From this analysis, we confirm that the peak at about 11 Hz is mainly due to the reverberation of the $P$ wave in the uppermost low-velocity layer. We also note that we can improve the fit of PTN at higher frequencies by adjusting the

![Figure 7. Comparison between observed PTN and synthesized PTN for the surface receivers. Symbols are the same as in Figure 4. Three curves mean synthesized PTN: the solid curve is the equipartition among both body and surface waves; the dashed curve is the equipartition among only surface waves; and the dotted curve is the equipartition among only body waves.](image-url)
velocity model in the shallow layers, which suggests a need to conduct inversion studies in the near future.

We emphasize again that equipartition of coda waves enables a simple modeling of PTN or H/V. This contrasts with the H/V method using ambient noise, the modeling of which is not straightforward because the constituents of noise wave fields are still actively debated (e.g., Bonnefoy-Claudet et al., 2005).

Interpretation of PTN in Terms of Seismic Interferometry

It is useful to make a parallel between the equipartition hypothesis and seismic interferometry (the retrieval of Green’s function in random wave fields). The reconstruction of Green’s function from the cross-correlation tensor of coda waves measured at two points was successfully demonstrated by Campillo and Paul (2003) and Paul et al. (2005) using data from Mexico and Alaska, respectively. Because autocorrelation and power spectral densities are Fourier transform pairs, we may anticipate a relation between energy partitioning in the coda and Green’s function. Independently, multiple scattering theory predicts that the ensemble averaged cross-correlation of fields measured at two points in a disordered medium is proportional to the imaginary part of (the ensemble average) Green’s function between these two points. In our analysis, no true ensemble average can be performed, and the theoretical argument does not apply directly. However, averaging over a large number of sources and time windows might allow us to use the equipartition hypothesis to find a relation between PTN and Green’s function.

To support this idea, we develop a simple argument based on the work of Weaver (1985) on the calculation of participations in diffuse fields. Let us assume that Earth is, on average, a layered medium over which random fluctuations are superimposed, as schematically illustrated in Figure 6a. As a consequence of the lateral fluctuations of the elastic properties, the mode of the background layered medium gets coupled in a random fashion, and we may apply the equipartition hypothesis (see equations 5–7). The power spectrum of the displacement field at position \( \mathbf{x} \) along direction \( i \), at angular frequency \( \omega \), and lapse time \( \tau \) in the coda can therefore be expressed as (Margerin, 2009)

\[
\Gamma_i(\mathbf{x}; t, \omega) = \int_{-\infty}^{\infty} \langle u_i(\mathbf{x}; t + \tau/2)u_i(\mathbf{x}; t - \tau/2)^* \rangle e^{i\omega\tau} d\tau \\
= \frac{\alpha^2(t)}{2\pi} \sum_n \delta(\omega - \omega_n) |\psi^*_n(\mathbf{x})|^2.
\] (9)

Figure 8. Comparison between observed PTN and synthesized PTN for the subsurface receivers. Symbols are the same as in Figure 5. Three curves mean synthesized PTN. The solid curve is the equipartition among both body and surface waves; the dashed curve is the equipartition among only surface waves; and the dotted curve is the equipartition among only body waves.

Figure 9. Comparison between observed PTN and synthesized PTN for the modified velocity structure at IWTH02 and IWTB02. Symbols are the same as in Figures 4 and 5. Three curves mean synthesized PTN. The solid curve is the equipartition among both body and surface waves; the dashed curve is the equipartition among only surface waves; and the dotted curve is the equipartition among only body waves.
In equation (9), the term \( \sigma^2(t) \) takes into account the slowly decreasing envelope of the coda. The sum \( \sum_n \) can be understood either as a sum over all modes of the Earth seen as a finite body or as a short-hand notation for the complete modal sum given in equation (4). The symbol \( \psi^i(x) \) denotes the eigenfunctions of the stratified Earth with eigenfrequencies \( \omega_n \), which are assumed to form a complete orthonormal set.

Equation (9) is to be compared to the imaginary part of the retarded Green’s function of the elastodynamic equation:

\[
\text{Im} G_{ii}(x, x; \omega) = -\sum_n \frac{\pi}{2\omega} \delta(\omega - \omega_n) |\psi^i_n(x)|^2 \quad (\omega > 0).
\]

(10)

This equation can be obtained from equations 1.25 and 1.27 in Economou (2006). Using our equations (9) and (10), we deduce the theoretical PTN at equipartition:

\[
\text{PTN}_{\text{UD}}(\omega, t) = \frac{\Gamma_s(x, t; \omega)}{\sum_i \Gamma_i(x, t; \omega)} = \frac{\text{Im} G_{ss}(x, x; \omega)}{\text{Tr}[\text{Im} G(x, x; \omega)]},
\]

(11)

where \( \text{Tr} \) denotes the trace of a tensor, and \( G \) is Green’s tensor. Thus, PTN at equipartition may be evaluated through numerical calculations of the elastodynamic Green’s function in the frequency domain. Equation (11) emphasizes the relation between local energy densities and an imaginary part of the Green’s function, as discussed in the context of seismic interferometry (Snieder et al., 2009).

To what extent PTN can truly be modeled by computing the simple Green’s function of a layered medium remains an open question. In the weak scattering regime (i.e., for weak disorder), the propagation properties of the Earth may be assumed to be relatively close to that of the idealized laterally averaged Earth. However, in many cases of interest, the simple assumption of an approximately layered Earth may be violated. Such will be the case for instance in the presence of interface topographies at the scale of the wavelength. This does not invalidate the general relation between Green’s function and energy partitioning of a diffuse field, but a more sophisticated Green’s function may be required.

It must be kept in mind that equation (11) may also break down if the coda wave field is imperfectly diffuse. For our dataset, equipartition holds in a strict sense up to frequencies of the order of 5 Hz only. At higher frequencies, it appears that a mixture of body waves suffices to fit simultaneously the observations at the free surface and at the borehole sensor.

**Conclusion**

We have studied the relative contributions of the horizontal and vertical components to the total kinetic energy of the S-wave coda from 60 local earthquakes using three borehole strong-motion stations in northeast Japan. For S-wave coda in lapse times of 40–80 s and in frequencies between 2 and 16 Hz, contribution of the sum of horizontal components is about 80% at frequencies lower than 5 Hz, and this proportion gradually increases with frequency at the surface receivers of the two rock sites. The surface receiver of the soft site shows that the horizontal energy is higher than 80% at frequencies lower than about 5 Hz but suddenly drops to about 20% around 10 Hz. At subsurface receivers located at a depth of about 100 m, the contribution of the sum of horizontal components is about 2/3 of that at low frequency and gradually increases with frequency, irrespective of site conditions. Using the method of Margerin (2009), we explained such observations by calculating the kinetic energy of random wave fields at equipartition in horizontally layered structures estimated from well logging. Through the modeling, we have tested three different assumptions of equipartition: (1) equipartition among body waves only, (2) equipartition among surface waves only, and (3) equipartition among both body and surface waves. None of the three assumptions makes a significant difference in explaining the observations at the surface receivers. On the other hand, the observations at the subsurface receivers are crucial to distinguish between the three assumptions: Assumption (3) is best at frequencies lower than about 5 Hz, while assumption (1) is best at higher frequencies. The decay of the surface-wave contribution at higher frequencies remains to be explained. Attenuation at shallower depths might be a cause, but we may also invoke other mechanisms related to the efficiency of body-to-surface mode conversions. The test has thus shown that the equipartition hypothesis holds for S-wave coda of local earthquakes. Based on this conclusion, the contribution of horizontal and vertical motions to the total kinetic energy, similar to H/V, of an S-wave coda serves as a useful tool for estimating subsurface layered velocity structures as an alternative or in conjunction with H/V methods using ambient noise. Thanks to equipartition, H/V can be modeled much more easily for S-wave coda than for ambient noise. Stability also constitutes a primary advantage of coda-based methods for site effect evaluation. It is noteworthy that subsurface receivers are crucially important to determine the constituents of the wave field.

**Data and Resources**

Seismograms and logging information used in this study can be obtained from the KiK-net, National Research Institute for Earth Science and Disaster Prevention, Tsukuba, Japan at http://www.kik.bosai.go.jp/kik/index_en.shtml (last accessed April 2011). The frequency characteristics of the KiK-net seismometers can be found at http://www.kik.bosai.go.jp/kik/ftp/pub/seismo/KiK_characteristics.pdf (last accessed April 2011). Figures were prepared using Generic Mapping Tools (Wessel and Smith, 1998).

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