Attenuation of S Waves in the Lithosphere due to Scattering by Its Random Velocity Structure

HARUO SATO

Department of Earth and Planetary Sciences, Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

Recent seismological studies suggest that the amplitude attenuation of S waves in the lithosphere may be primarily due to scattering by its random inhomogeneity. There is, however, a disturbing discrepancy between statistical scattering theory and observation with regard to the asymptotic frequency dependence of \( Q^{-1} \) in the high-frequency limit. Although the observed \( Q^{-1} \) decreases with frequency above 1 Hz, the usual mean wave formalism predicts \( Q^{-1} \) of media with random velocity fluctuation increases with frequency; we take a mean over waves with large travel time fluctuations that are caused by long-scale velocity fluctuations compared with the wavelength studied. We propose a new statistical averaging method: mean wave is defined after the correction of travel time fluctuations. The random media with a mean square fractional velocity fluctuation of \( 5.0 \times 10^{-3} \) and a correlation distance of 4.0 km will explain the observed and partially conjectured \( Q^{-1} \) for frequencies from 0.05 to 30 Hz.

INTRODUCTION

The \( Q^{-1} \) value of S waves in the earth's lithosphere has been found to be of the order of \( 10^{-3} \sim 10^{-2} \) and to decrease with frequency above 1 Hz by the use of a single-station method in which coda wave spectra were used to eliminate the source effect [Rautian and Khalturin, 1978; Aki, 1980a; Sato and Matsumura, 1980; Roecker, 1981]. Fedotov and Boldyrev [1969] obtained \( Q^{-1} \) of S waves using a single-station method with more restrictive assumptions on the frequency dependence of \( Q^{-1} \) and the source spectrum estimation. We illustrate this observed frequency dependence of \( Q^{-1} \) of S waves in Figure 1. Combining the observed frequency dependence with the fact that \( Q^{-1} \) for frequencies lower than 0.05 Hz is less than \( 2 \times 10^{-3} \) inferred from the attenuation of long-period surface waves, Aki [1980a] conjectured that \( Q^{-1} \) of S waves had a peak around 0.5 Hz. We can roughly illustrate this by the region surrounded with a dotted line in Figure 1.

The lithosphere appears to be the most heterogeneous region of the earth. We are especially interested in the inhomogeneity of the velocity structure. We can easily see small-scale inhomogeneity in acoustic velocity well log data [Suzuki et al., 1981] and large-scale inhomogeneity in the structure determined by a three-dimensional inversion method [Aki et al., 1976]. The whole spectrum of structure of the lithosphere cannot be deterministically measured in detail. A statistical modeling is required for its description.

We know that waves generally attenuate with propagation distance in randomly inhomogeneous media. Energy loss by scattering due to the random structure of the lithosphere has been recently studied as an amplitude attenuation mechanism of seismic waves [Beaudet, 1970; Sato, 1979; Aki, 1980a, b; Wu, 1982]. There are statistical scattering theories: the mean wave formalism [Frisch, 1968; Howe, 1971] and the Born approximation [Chernov, 1960]. However, these theories predict that the \( Q^{-1} \) value, for waves in media with random fractional velocity fluctuation, increases monotonously with frequency in the high-frequency limit in wave-lengths shorter than the correlation distance. Both theories fail to explain the observed decreasing behavior of \( Q^{-1} \) for S waves with frequency above 1 Hz.

Aki [1980a] found that the direct application of the Born approximation to seismological \( Q^{-1} \) measurement was inadequate. In the seismological \( Q^{-1} \) measurement, we measure only the maximum amplitudes of S wave phase on band-pass-filtered seismograms. Forward scattered S waves, which are strong in the high-frequency limit, may arrive nearly simultaneously with the primary S waves because they have close propagation directions and close propagation velocities. Practically, we should not consider that the energy loss due to forward scattering is observed. Regarding scattering into only the backward half space as energy loss, Wu [1982] calculated \( Q^{-1} \) for scalar waves; the \( Q^{-1} \) value obtained decreased with frequency in the high-frequency limit.

Recently, Sato [1982, hereinafter referred to as paper 1] clarified the reason why the \( Q^{-1} \) value increased with frequency in the high-frequency limit from the point of view of the mean wave formalism. He showed the inadequacy for the direct application of the usual mean wave formalism to the amplitude attenuation measurement of impulsive waves such as seismic waves and proposed a new, more appropriate statistical averaging method.

In this paper, we first briefly describe this new statistical averaging method in relation to the seismological \( Q^{-1} \) measurement on the basis of the scalar model. Applying the \( Q^{-1} \) value obtained by the new method to the conjectured frequency-dependent \( Q^{-1} \) value for S waves in the lithosphere, we evaluate its randomness quantitatively. Next, we compare the results obtained with different kinds of measurements for the random structure of the lithosphere.

SCALAR WAVES IN RANDOM MEDIA

For simplicity, we investigate the scalar wave propagation in random media. A scalar wave \( \phi(x, t) \) at space coordinate \( x \) and time \( t \) in a three-dimensional inhomogeneous medium is
Sato: Attenuation due to Scattering by Random Structure

**Q** of S waves in the Lithosphere

Fig. 1. Frequency dependence of **Q**-1 for S waves in the lithosphere measured by a single-station method. Solid circles, Southern Kuril Island [Fedotov and Boldyrev, 1969]; open circles, Iwatsuki, Kanto, Japan [Sato and Matsumura, 1980]; open squares, Dodaira, Kanto, Japan [Aki, 1980a]; and bold line, Garm, Central Asia [Rautian and Khalturin, 1978].

governed by

\[ \{ \nabla^2 - (1/C^2)[1 - 2\xi(x)] \} \Phi(x, t) = 0 \]  

(1)

where \( C \) is the mean wave velocity and \( \xi(x) \) is the fractional velocity fluctuation, that is, the lithosphere is modeled by a continuous random medium.

We remember that modeling the S wave propagation in the lithosphere by the scalar wave equation is too simple; we neglect SP conversion scattering and effects of spatial derivative of elastic coefficients. The principal nature of amplitude attenuation in random media, however, will be mathematically well described by this simplest modeling.

We assume that \( \xi(x) \) is a homogeneous (stationary) and isotropic random function of the space coordinate \( x \) [Batchelor, 1953]. Let \( \xi \) be a first-order small quantity (\( |\xi| \ll 1 \)). Now, we imagine a family of random media, that is, an ensemble of \( \xi(x) \). An ensemble average of \( \xi \) must be zero:

\[ \langle \xi(x) \rangle = 0 \]  

(2)

where the angle brackets represent an ensemble average.

The autocorrelation function of the fractional velocity fluctuation \( \xi \) is defined by

\[ R(u) = \langle \xi(x + u) \xi(x) \rangle \]  

(3)

where \( R \) is a function of \( u = |u| \) because of the homogeneity and the isotropy properties of \( \xi(x) \). The autocorrelation function is mainly characterized by the mean square fractional velocity fluctuation \( \xi^2 = R(0) \) and the correlation distance \( a \), where \( R(u) \) is much smaller than \( \xi^2 \) for \( u > a \). The power spectral density function of the fractional velocity fluctuation \( \xi \) is defined by a Fourier transformation of \( R \):

\[ P(m) = \int \int \int R(u) e^{-im\cdot u} \, du, \]  

(4)

where \( m \) is a wave number vector and \( P \) is a function of \( m = |m| \). Both the autocorrelation function \( R \) and the power spectral density function \( P \) are used to characterize the random media rather than the fluctuation \( \xi \).

**Comparison of Statistical Averaging Methods**

In the mean wave formalism, we imagine \( N \) experiments of impulsive wave propagation through random media; waves \( \Phi \) are generated under an identical initial condition, when the duration time is assumed to be one or two times as long as the characteristic period. The mean wave \( \langle \Phi \rangle \) is defined as an ensemble average of waves \( \Phi \). We schematically illustrate a temporal change in waves in each medium in the high-frequency limit (\( ak \gg 1 \)) in Figure 2a, where \( k \) is the wave number under consideration. If the mean square fractional velocity fluctuation \( \xi^2 \) is small, the attenuation of the maximum amplitude in each experiment will be also small. For \( ak \ll 1 \), travel time fluctuations are small compared to the wavelength studied; therefore, the maximum amplitude of the mean wave \( \langle \Phi \rangle \) will show a small attenuation. Travel time fluctuations become comparable to or larger than the wavelength \( \lambda \) studied as \( ak \) increases. In spite of the small amplitude attenuation in each experiment, the maximum amplitude of the mean wave \( \langle \Phi \rangle \) will strongly decrease in the high-frequency limit, as illustrated in Figure 2a, because of large travel time fluctuations. This is the explanation of why \( \xi^2 \) increases with frequency in the high-frequency limit.

In the seismological \( \xi^2 \) measurement, however, we don't take an ensemble average of waves, as schematically illustrated in Figure 2a. We usually take an ensemble average of their maximum amplitudes irrespective of their travel time.
fluctuations at any hypocentral distance. We calculate $Q^{-1}$ from the gradient of the regression line for the plots of the maximum amplitude versus hypocentral distance. Therefore, the usual mean wave formalism is not applicable to the amplitude attenuation measurement of impulsive waves such as seismic waves, especially in the high-frequency limit.

We may say that impulsive waves are averaged after shifting in time so as to arrange the wave peaks in a line, as illustrated in Figure 2b, irrespective of travel time fluctuations; the travel time fluctuations are caused by the longer-wavelength component of the velocity fluctuation compared with the characteristic wavelength $\lambda$ of the impulsive wave studied. The maximum amplitude of the newly averaged wave $\langle \phi \rangle$ shown at the bottom of Figure 2b will probably not decrease so strongly as that of the usual mean wave $\langle \phi \rangle$ previously shown in Figure 2a even in the case of $ak >> 1$.

**TRAVEL TIME CORRECTED MEAN WAVE FORMALISM**

It is difficult to define mathematically rigorously this averaging method; therefore, we introduce a new wave $\psi$ in which the travel time fluctuation $\delta t^L$ is corrected (paper 1):

$$\psi(x, t) = \frac{1}{2}[\psi(x, t + \delta t^L(x))]$$

(5)

We suppose that only the fractional velocity fluctuation with wavelength longer than $2\lambda$ causes the travel time fluctuation because such a wavelength can sample the fractional velocity fluctuation with an identical sign over the wavelength $\lambda$. The velocity with wavelength shorter than $2\lambda$ loses the real meaning of the wave propagation velocity. We may say that the boundary wavelength of $2\lambda$ gives the lower bound of $Q^{-1}$ for the same random fluctuation. If we take a boundary wavelength longer than $2\lambda$, we can easily show that the corresponding $Q^{-1}$ is larger than $Q^{-1}$ for the boundary wavelength of $2\lambda$. We define the mean wave $\langle \psi \rangle$ according to the usual averaging method. We regard the attenuation of the mean wave $\langle \psi \rangle$ as the amplitude attenuation of impulsive waves. We call this method 'a travel time corrected mean wave formalism.' We may say that this new averaging method is more adequate to the seismological $Q^{-1}$ measurement than the usual mean wave formalism.

Now, we have to define the travel time fluctuation $\delta t^L(x)$ rigorously. We first write the fractional velocity fluctuation $\xi(x)$ in a Fourier integral form as

$$\xi(x) = \frac{1}{(2\pi)^3} \int \int \int \xi(m)e^{imx} dm$$

(6)

where $m$ is a wave number vector and $\xi(m)$ is the fractional velocity fluctuation in wave number vector space. We decompose the fractional velocity fluctuation $\xi$ into two parts, as schematically shown in Figure 2c: the shorter-wavelength component $\xi^S$ and the longer-wavelength component $\xi^L$ as

$$\xi(x) = \xi^S(x) + \xi^L(x)$$

(7)

$$\xi^S(x) = \frac{1}{(2\pi)^3} \int \int \int \xi(m)e^{imx} H\left(m - \frac{k}{2}\right) dm$$

(8)

$$\xi^L(x) = \frac{1}{(2\pi)^3} \int \int \int \xi(m)e^{imx} \left[1 - H\left(m - \frac{k}{2}\right)\right] dm$$

(9)

where $k = 2\pi/\lambda$ is the wave number, $m = |m|$, $k/2$ is the boundary wave number, and $H(m)$ is the step function defined by

$$H(m) = \begin{cases} 1 & m \geq 0 \\ 0 & m < 0 \end{cases}$$

(10)

The travel time fluctuation $\delta t^L$ caused by the longer-wavelength component of the fractional velocity fluctuation $\xi^L$ is written as an integral along the ray path of the waves:

$$\delta t^L(x) = \frac{1}{(C)} \int_{\text{ray path}} \left\{1 - 11 - 2\xi^L(x')^{10}\right\} ds(x')$$

(11)

where $ds$ is an infinitesimal line element along the ray path and terms of an order higher than the second power of $\xi^L$ are neglected. A small change in the direction of the ray path due to $\xi^L$ is neglected.

Let us consider wave propagation along the $X_1$ axis. We can rewrite (11) in differential forms as

$$\partial_{x_1} \delta t^L(x) = \frac{1}{C} \xi^L(x)$$

(12)

Substituting (5) into (1) and using (12), we obtain the wave equation for $\psi(x, t)$. Let us consider a monochromatic wave $\psi_0(x)$ of angular frequency $\omega_0$:

$$\psi(x, t) = \psi_0(x)e^{-i\omega_0 t}$$

(13)

The wave equation for $\psi_0$ is written as

$$(\nabla^2 + k_0^2)\psi_0 = 2k_0^2 \xi^S \psi_0 + 2k_0^2 \xi^L \psi_0 + 2ik_0 \xi^L \partial_{x_1} \psi_0 + ik_0 \partial_{x_1} \xi^L \psi_0$$

(14)

where terms of an order higher than the second power of $\xi^S$ and $\xi^L$ are neglected. The first and the second terms show the effect of the fractional velocity fluctuations. The third and the fourth terms show the correction of the travel time fluctuation. We easily find that the second and the third terms cancel each other out in the case of a plane wave, substituting $\exp(ik_0 x_1)$ into $\psi_0$.

We solve (14) adopting the binary interaction approximation in the mean wave formalism; it may also be called the first-order smoothing method, a general method for treating such a stochastic wave equation. This approximation is equivalent to the simplest approximation based on the Dyson equation in the formal perturbation method, well known as the renormalization procedure in theoretical physics, which retains only the first term of the mass operator [Frisch, 1968]. The validity of the mean wave formalism does not require any restriction on dimensions of the region occupied by the random inhomogeneities.

We decompose the wave $\psi_0$ into the mean wave $\langle \psi_0 \rangle$ and the fluctuating wave $\psi_0'$ as

$$\psi_0 = \langle \psi_0 \rangle + \psi_0'$$

(15)

where

$$\langle \psi_0' \rangle = 0$$

(16)
Assuming that the fluctuating waves \( \phi_0' \) are created from the first-order interaction between \( \phi_0 \) and \( \xi \) and \( \xi' \), we obtain the binary interaction approximation equation for the mean wave \( \phi_0' \), where the effects of the random fractional velocity fluctuation are included up to the second-order small quantity. Substituting a plane wave solution propagating along the \( X_1 \) axis into \( \phi_0' \), we obtain the dispersion relation for \( \phi_0' \). Supposing a small deviation from a linear dispersion relation, we can evaluate the \( Q^{-1} \) value from the imaginary part of the dispersion relation: the \( Q^{-1} \) value characterizes the amplitude attenuation of a mean wave per unit cycle as \( \exp (-\pi Q^{-1}) \). Detailed derivation is given in paper I. We have

\[
Q^{-1}(k_0) = \frac{k_0^3}{2\pi} \int_0^\pi \frac{P}{2k_0} \sin \left( \frac{\theta}{2} \right) \left\{ H(\theta - \theta_c) + \sin^4 \left( \frac{\theta}{2} \right) \sin \theta \right\} d\theta
\]

where \( \theta \) is a scattering angle measured from the \( X_1 \) axis, which is the propagation direction of the mean waves, and the critical angle \( \theta_c = 2 \sin^{-1}(1/4) = 29^\circ \). If we substitute \( 1 \) into the braces in (17), we get the \( Q^{-1} \) value of the usual mean wave formalism and the Born approximation. We find out that the correction of the travel time fluctuations gives the strong decreasing factor \( \sin^4 \left( \frac{\theta}{2} \right) \) in the integral inside of a cone around the forward direction. This factor prevents \( Q^{-1} \) from increasing with frequency in the high-frequency limit.

### The \( Q^{-1} \) Value for the von Karman Autocorrelation Function

We calculate \( Q^{-1} \) for the von Karman autocorrelation function, which is the most typical example for random fractional velocity fluctuation; it is widely used in the turbulence theory [Tatarsky, 1961]. The von Karman autocorrelation function of order \( \nu \) is given by

\[
R_v(u) = \frac{e^{2}}{2\pi^2\Gamma(\nu)} \left( \frac{u}{\alpha} \right)^{\nu} K_\nu \left( \frac{u}{\alpha} \right) \quad 0 < \nu \leq \frac{1}{2}
\]

where \( \Gamma(\nu) \) is the gamma function and \( K_\nu \) is the Bessel function of the second kind of imaginary argument of order \( \nu \) (see Figure 3a). The exponential autocorrelation function is given by the case of order 1/2. The corresponding power spectral density function for the von Karman autocorrelation function of order \( \nu \) is given by

\[
P_v(m) = \frac{e^{2}(2\pi^2a^3)}{\Gamma(\nu)(1 + a^2m^2)^{\nu+3/2}}
\]

In the high-frequency limit (\( \alpha m \gg 1 \)), the power spectral density function decreases with frequency of negative power \(-2\nu - 3\). Short-wavelength components become much more contained in the fractional velocity fluctuation as the order \( \nu \) decreases.

Substituting (19) into (17), we obtain the \( Q^{-1} \) value as

\[
Q^{-1}_v(k_0) = \frac{2\pi^3}{16k_0^2} \left[ \frac{1}{1 + a^2k_0^2/4} - \frac{4}{a^2k_0^2} \ln \left( 1 + a^2k_0^2/4 \right) \right] \quad \nu = \frac{1}{2}
\]

where the subscript \( \nu \) corresponds to the order. In the limiting case, we obtain

\[
Q^{-1}_v(k_0) = \frac{2\pi^3}{16k_0^2} \left[ \frac{1}{1 + a^2k_0^2/4} - \frac{4}{a^2k_0^2} \ln \left( 1 + a^2k_0^2/4 \right) \right] \quad \nu = \frac{1}{2}
\]

Each \( Q^{-1}_v \) has a peak of the order of \( e^2 \) around \( \alpha k_0 \sim 2 \), the wavelength of \( \pi \) times as long as the correlation distance. The peak value becomes larger and the peak frequency becomes lower as the order \( \nu \) increases. In the low-frequency limit (\( \alpha k_0 \ll 1 \)), every \( Q^{-1}_v \) is proportional to the third power of frequency: the Rayleigh scattering. In the high-
Moreover, we compare the magnitude of mean square fractional velocity fluctuations and the length of correlation distances estimated from several other different kinds of methods. One method is the teleseismic $P$ wave analysis based on the Chernov [1960] theory, where inhomogeneities are estimated from both amplitude and phase fluctuations. Analyzing the records obtained at LASA in Montana, Aki [1973] gave the estimate of $\varepsilon^2$ and $a$ for $P$ wave velocity to be $1.6 \times 10^{-2}$ and 10 km, and Capon [1974] gave those values to be $4 \times 10^{-4}$ and 12 km, respectively. The analysis only using phase fluctuations gave the estimation of $\varepsilon^2$ considerably smaller than that estimated from the analysis using both the fluctuations. [Berteussen et al., 1975]. Analyzing the records of teleseismic $P'P'$ phase observed at NORSAR in Norway based on a method similar to the Chernov theory, Vinik [1981] estimated $\varepsilon^2$ and $a$ to be $2.56 \times 10^{-4}$ and 13 km, respectively.

Aki et al. [1976] applied the three-dimensional inversion method to many $P$ time readings of the teleseismic records obtained at LASA, and they found out that the mean square of the true $P$ wave fractional velocity fluctuation was $0.9 \times 10^{-2}$; this value should be interpreted as the lower bound of $\varepsilon^2$ because such a travel time analysis measures only the longer-wavelength component of the velocity fluctuation. And an average distance between the high- and the low-velocity peaks was estimated to be several tens of kilometers; this value is longer than the real correlation distance.

Acoustic velocity well log gives us a deterministic knowledge about the one-dimensional vertical inhomogeneity of the upper limb of the lithosphere. Suzuki et al. [1981] estimated the values of $\varepsilon^2$ and $a$ from the well log in the pre-tertiary formation in the Kanto district, Japan; $\varepsilon^2$ and $a$ were $5 \times 10^{-3}$ and 30 m in crystalline schist and quartz porphyry at Iwatsuki, $5 \times 10^{-3}$ and 15 m in crystalline schist at Shimohsa, $10^{-2}$ and 30 m in shale and sandstone at Fuchu, respectively. Wu [1982] estimated the values of $\varepsilon^2$ and $a$ to be $1.44 \times 10^{-4}$ and 1.5 m, respectively, from the well log in granodiorite at Fenton Hill in New Mexico. The correlation distances were estimated to be considerably smaller than the real one because we subtracted long-wavelength components of velocity fluctuation from the original log data and the total sample lengths were short.

We plot the values of $\varepsilon^2$ and $a$ estimated from various kinds of measurements with the present results in Figure 4, irrespective of the difference between $S$ and $P$ wave velocity fluctuations. We find our estimation of $a$ is smaller than the values estimated from the teleseismic $P'P'$ wave analyses at LASA. Our estimation of $\varepsilon^2$ is comparable to the values estimated from well logs in Japan and larger than the values estimated from the teleseismic $P$ wave analyses by the Chernov theory and by the three-dimensional inversion method at LASA.

### CODA Wave Excitation

We can theoretically evaluate the angular dependence of intensity of scattering due to random fractional velocity fluctuation on the basis of the Born approximation. First, we imagine harmonic plane waves incident upon random media when the randomness is supposed to extend over a relatively small region of space but to be larger than the correlation distance. According to Aki [1980b], we define the directional scattering coefficient $g(\theta)$, that is, 4π times as large as the ratio of scattered waves' energy to the primary plane waves'
energy in the far field per unit travel distance of the primary waves and per unit solid angle at a scattering angle \( \theta \).

We have

\[
g(\theta) = \left( k_0^4 / 4 \right) P \left[ 2 k_0 \sin \left( \theta/2 \right) \right]
\]  

(24)

where \( k_0 \) is the wave number of the primary plane waves.

Coda wave excitation is the most typical scattering phenomenon of seismic waves. We can estimate the intensity of backward scattering \( g(\pi) \) from the coda wave excitation analysis, supposing that S wave energy is radiated spherically from the source and that coda waves are single scattered S waves by scatterers distributing homogeneously and isotropically in three-dimensional space [Aki and Cheuet, 1975; Sato, 1977].

For the von Karman autocorrelation function of order \( \nu \), substituting (19) into (24), we obtain the directional scattering coefficient at \( \theta = \pi \) as

\[
g(\pi) = \frac{8 \pi^{1/2} e^2 a^2 k_0 \Gamma(\nu + 3/2)}{\Gamma(\nu)(1 + 4 a^2 k_0^2)\pi^{3/2}}
\]  

(25)

We illustrate theoretical curves given by (25) for the five sets of parameters \( (\nu, e^2, a) \), estimated from \( Q^{-1} \) of S waves, with the estimate from the coda wave excitation analysis by Sato [1978] and Aki [1980b] in the Kanto district of Japan in Figure 5. Sato estimated \( g(\pi) \) in the range from \( 4 \times 10^{-3} \) to \( 4 \times 10^{-2} \) \( \text{km}^{-1} \) for the frequency range from 1 to 30 Hz and Aki estimated it in the range from \( 10^{-2} \) to \( 4 \times 10^{-2} \) \( \text{km}^{-1} \) around 1.5 and 3 Hz. Five theoretical curves appear to be a little smaller than the observed values; \( e^2 \) estimated from \( Q^{-1} \) of S waves is a little too small to explain the observed coda wave excitation. We remember that the value of \( e^2 \) estimated from \( Q^{-1} \) is the upper bound of it. This discrepancy may be caused by rough measurements of coda wave excitation, or it may suggest the necessity for reconstruction of the coda wave excitation model. We may have to consider anisotropic and inhomogeneous distribution of scatterers, anisotropic scattering of vector waves due to three independent fluctuations in random elastic media, and moreover, multiple scattering.

**DISCUSSION**

We may say that seismological attenuation measurement statistically measures mainly the shorter-wavelength component of the fractional velocity fluctuation compared with the wavelength studied. This is complementary to the three-dimensional inversion method that deterministically measures the longer-wavelength component of the fractional velocity fluctuation; only the arrival time of initial motion is carefully read on the seismogram, but no attention is paid to the wave shape itself. We note that Mereu and Ojo [1981] recently made numerical experiments on travel time fluctuation in random velocity structure on the basis of the ray theory.

The correction of the travel time fluctuations is shown to be equivalent to excluding the scattering energy loss inside of a cone around the forward direction. The \( Q^{-1} \) value, corresponding to the von Karman autocorrelation function for the fractional velocity fluctuation of order 1/3 with \( e^2 = 5 \times 10^{-3} \) and \( a = 4.0 \) km, quantitatively explains the observed and partially conjectured frequency dependence of \( Q^{-1} \) of S waves in the lithosphere for frequencies from 0.05 to 30 Hz; we consider scattering due to only S wave velocity fluctuation and neglect SP conversion scattering. The value of \( e^2 \) estimated should be interpreted as the upper bound of it. The values of \( e^2 \) estimated are comparable to those estimated from the acoustic velocity well log in the Kanto district of Japan and larger than those estimated from the teleseismic P wave analyses at LASA. The values of \( a \) estimated are a little smaller than those estimated from the teleseismic P wave analyses. The intensity of backward scattering, calculated theoretically from the inhomogeneity obtained above,

![Fig. 5. Sato(1978) Coda Waves in Kanto](image-url)
appears to be a little smaller than those estimated from the coda wave excitation analyses in the Kanto district.

We need more complete measurement of $Q^{-1}$ of $S$ waves in frequencies lower than 1 Hz and more precise measurement of the intensity of backward scattering in the coda wave excitation analysis. Of course, we need measurement of $Q^{-1}$ of $P$ waves, too. And we suggest the regional study of the inhomogeneity in the lithosphere in relation to the tectonic activity.

We simply applied the theory of $Q^{-1}$ for scalar waves to the observed $Q^{-1}$ for $S$ waves in this preliminary study. We have to construct an amplitude attenuation theory for vector waves in random elastic media, when not only $P$ and $S$ wave velocity fluctuations and their spatial derivatives but also density fluctuation contributes to the attenuation. Then, the magnitude of $S$ wave fractional velocity fluctuation will become smaller than that of the present result to explain the observed $Q^{-1}$ of $S$ waves because of the presence of three independent fluctuations. The present author believes that the correction of the travel time fluctuation will prevent the $Q^{-1}$ value from increasing with frequency in the high-frequency limit even in the case of vector waves. Furthermore, it may be more realistic to adopt a horizontal correlation distance different from a vertical correlation distance.

We have shown the inadequacy of the direct application of the usual mean wave formalism to the seismological $Q^{-1}$ measurement and constructed a new statistical averaging method appropriate for it; however, we note that there may be a way to measure the inhomogeneity of the lithosphere according to the usual mean wave formalism.

Acknowledgments. The author wishes to thank K. Aki for his encouragement, criticism, and valuable discussions. The author is also grateful to S. Kinoshita for his discussions on well log data. This work was supported by the National Science Foundation under grant 8005720 PFR.

References

Aki, K., Attenuation of shear waves in the lithosphere for frequencies from 0.05 to 25 Hz, Phys. Earth Planet. Inter., 21, 50–60, 1980a.

(Received December 15, 1981; revised June 4, 1982; accepted June 11, 1982.)