Numerical Basis of the Separation of Scattering and Intrinsic Absorption from Full Seismogram Envelope
— a Monte-Carlo Simulation of Multiple Isotropic Scattering —

by

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Abstract

Seismic wave attenuation is caused by two major factors, scattering attenuation and intrinsic absorption. A method to separate these two factors from total attenuation is proposed for a case where scattering is isotropic and the random distribution of scatterers and that of absorbers are uniform. The seismic waves recorded at a station from an earthquake can be divided into three portions: a direct wave plus an earlier part of coda waves, a middle part of coda waves and a later part of coda waves. The latter two portions are composed of scattered waves only. The time integral of energy density of each portion is numerically simulated based on the Monte-Carlo method, and is plotted against hypocentral distance. The curves of integrated energy are very sensitive to the seismic albedo which is the ratio of scattering loss to the total attenuation. We offer a set of curves of the integrated energy vs. hypocentral distance for different values of the seismic albedos and different strengths of total attenuation. These curves make it possible to evaluate separately the scattering strength and intrinsic absorption by comparison of the time integrals of actually observed band-pass filtered seismogram’s power with the simulated curves.

1. Introduction

Seismic wave attenuation is caused by two major factors, scattering by the heterogeneity in the medium and intrinsic absorption by anelasticity of the medium. The quality factor $Q_s$, which represents an attenuation strength, is given by

$$Q_{s}^{-1} = Q_{i}^{-1} + Q_{l}^{-1}$$

where $Q_s$ is the quality factor due to scattering and $Q_l$ is that of intrinsic absorption. (Note that we pay attention in this paper

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only to S-wave and S-coda, not to P-wave and P-coda. Symbols \( Q_s \), \( Q_s' \), and \( Q_i \) in this paper represent quality factors of S-wave. Intrinsic absorption converts seismic energy into heat, but scattering merely changes the direction of wave propagation. \( Q_i' \) can be estimated from observations of the decay rate of direct wave amplitude with distance. Dainty (1981) tried to separate \( Q_i' \) and \( Q_s' \) from \( Q_i' \) assuming frequency independence in \( Q_i' \). But there is no physical reason to assume frequency independence in \( Q_i' \). It is impossible to separate them from the \( Q_i' \) measured from direct wave energy without \textit{a priori} assumption of frequency dependence. Coda waves are considered to be composed of scattered waves. Analysis of both direct waves and coda waves has been considered as one approach for estimating scattering strength, \( Q_s' \).

Attempts to separate the two factors, \( Q_s' \) and \( Q_i' \), from \( Q_i' \) have been made and several scattering models for coda envelope shape have been proposed. Although the single scattering model (Aki, 1969; Aki and Chouet, 1975; Sato, 1977) makes a good explanation of the way how coda waves are excited, this model is valid only for the case where the lapse time measured from the origin time is much shorter than the mean free time. The diffusion model (Wesley, 1965; Aki and Chouet, 1975) is valid only for the case where the lapse time is much longer than the mean free time. We need a multiple scattering model to interpolate between the above two extreme models. Kopnichev (1977) formulated a multiple scattering model considering the double and triple scattering when scattering is isotropic, and Gao \textit{et al.} (1983) considered up to seventh-order isotropic scattering. Unfortunately the formula derived by Gao \textit{et al.} (1983) is valid only for the case in which the source and the receiver are located at the same position. Thus, it may be difficult to use the formula of Gao \textit{et al.} (1983) for the separation of \( Q_s' \) and \( Q_i' \) in practical analysis. Frankel and Wennerberg (1987) attempted to separate these two values from coda \( Q_i' \), which represents the decay rate of the coda amplitude with time, using their energy flux model which presupposes the uniform distribution of scattered energy. They concluded that coda \( Q_i' \) found from the single scattering model is nearly equal to \( Q_i' \). There is no description of the physical mechanism of scattering in their model.

Wu (1985) formulated the energy density distribution of multiple scattered seismic waves including the direct wave in the stationary state based on the radiative transfer theory (Ishimaru, 1978). This formula analytically predicts the spatial distribution of the integral of seismic wave power over an infinite time length. Wu's approach modeled the spatial distribution of seismic wave energy, while former authors (Gao \textit{et al.}, 1983; Frankel and Wennerberg, 1987) paid attention to the temporal change of energy. Wu's (1985) theory was applied to data in the Hindu Kush region (Wu and Aki, 1988), in the northeast United States and Canada (Toksöz \textit{et al.}, 1988), in California (Mayeda \textit{et al.}, 1991), and in Alaska (McSweeney \textit{et al.}, 1991), for the estimation of \( Q_i' \) and a parameter \( B_0 = Q_s' / Q_i' \), called the seismic albedo. However, they all used a finite length lapse time window for analyzing the data since it is practically impossible to observe a seismogram to infinite lapse time. For example, Wu and Aki (1988) used 32 s for the time window.

Recently Hoshiba (1991) numerically simulated the space-time distributions of the multiple scattered seismic wave energy for the case where the source and the receiver are spatially separated. He supposed that point-like isotropic scatterers are randomly and uniformly distributed. Results obtained by the simulation are in agreement with an integral form solution of the same problem by Zeng \textit{et al.} (1991); we note that numerical integration is necessary to evaluate the energy density even for Zeng \textit{et al.} 's formulation. Integral of the energy density obtained by the simulation
for a very long lapse time window also agrees well with Wu’s (1985) result. Hoshiba (1991), however, pointed out that the discrepancy between a finite time window of observation and an infinite time window of Wu’s theoretical model is serious. \( Q_{r}^{t} \) and \( B_{0} \) estimated by Wu and Aki (1988), Toksöz et al. (1988), Mayeda et al. (1991) and McSweeney et al. (1991) may have some errors.

In our new method described in the following sections, both spatial and temporal changes of direct wave energy and coda wave energy, and also the finiteness of lapse time window are taken into account. Therefore our method is the most useful and exact at this stage for the separation of these two factors. Comparing the observed energy distributions with those by numerical simulation (Hoshiba, 1991), we can estimate the most appropriate values for the two parameters, \( Q_{r}^{t} \) and \( B_{0} \).

The main purpose of this paper is to describe the method to separate \( Q_{s}^{t} \) and \( Q_{r}^{t} \) from \( Q_{r}^{t} \) and to present the plots of energy distributions against hypocentral distance predicted by simulation. An example of application of our method will be briefly described in section 5.

2. Simulation of the energy density of multiple scattered waves

The following are the basic assumptions for the numerical simulation:

1. S waves are radiated spherically from a point-like source with a very short duration.

2. There is no conversion scattering between P and S waves; S wave energy travels with a constant velocity \( v \).

3. There is no wave conversion between different frequencies.

4. Point-like scatterers with effective scattering cross section \( \sigma_{s} \) are randomly distributed with a uniform density \( n_{0} \).

5. Scattering is isotropic and the scattering power per unit volume is characterized by scattering coefficient \( g \equiv \sigma_{s} n_{0} \). That is, scattering attenuation \( Q_{s}^{t} = gu/\omega \), where \( \omega \) is the angular frequency of the S wave.

6. There is intrinsic absorption \( Q_{r}^{t} \), which is spatially uniform. We will introduce non-zero \( Q_{r}^{t} \) for the simulation in the next section, but we simulate the case of \( Q_{r}^{t} = 0 \) in this section.

The Monte-Carlo simulation method is adopted to obtain the energy distribution of a multiple scattered seismic wave. The trajectory of many particles represents the propagation of S-wave energy. First the energy density for each order of scattered wave is calculated separately, then these energy densities are summed to obtain the energy density including the multiple scattering effects.

The following procedures are taken to obtain the energy density of the \( n \)-th order scattered wave as schematically shown in Fig. 1. Let \( s \), \( \theta \) and \( \phi \) represent the spherical coordinates. Particles are shot from the source, \( \mathbf{R}_{0} \). Let \( \mathbf{R}_{1} \) represent the first scattering point, and \( \mathbf{R}_{k} \) \((k \geq 1) \) the \( k \)-th order scattering point. The location \( \mathbf{R}_{k} \) is determined from \( \mathbf{R}_{k-1} \) by using the vector \( \mathbf{r}_{k} = (s_{k}, \phi_{k}, \theta_{k}) \),

\[
\mathbf{R}_{k} = \mathbf{R}_{k-1} + \mathbf{r}_{k}, \tag{1}
\]

where \( s_{k}, \theta_{k} \) and \( \phi_{k} \) are mutually independent random variables according to probability,

\[
P(0 \leq s_{k} \leq s_{0}, 0 \leq \theta_{k} \leq \theta_{0}, 0 \leq \phi_{k} \leq \phi_{0}) = \frac{\theta_{0}}{2\pi} \frac{1}{2} (1 - \cos \phi_{0}) \cdot [1 - \exp(-gs_{0})], \tag{2}
\]

in ranges of \([0, \infty)\), \([0, 2\pi]\) and \([0, \pi]\), respectively. This probability for \( \theta_{k} \) and \( \phi_{k} \) is derived from the spherical radiation of the particles from the source \((k = 1)\) and for isotropic scattering \((k \geq 2)\). From the definition of \( s_{k} \), \( g \) is the reciprocal of the mean free path.

The probability that a particle scattered at \( \mathbf{R}_{k} \) passes through a unit area
around the receiver \( \mathbf{R}_r \), perpendicular to the vector \( \mathbf{R}_r - \mathbf{R}_n \), is given by

\[
P \left( z \geq \left| \mathbf{R}_r - \mathbf{R}_n \right| \right) = \frac{\Delta \theta}{2\pi} \cdot \frac{\sin \phi}{2} \cdot \Delta \phi \cdot \left[ 1 - \left( 1 - \exp\left( -g \left| \mathbf{R}_r - \mathbf{R}_n \right| \right) \right] \\
= \frac{1}{4\pi \left| \mathbf{R}_r - \mathbf{R}_n \right|^2} \cdot \exp\left( -g \left| \mathbf{R}_r - \mathbf{R}_n \right| \right),
\]

(3)
since

\[
\text{(unit area)} = 1 = \left| \mathbf{R}_r - \mathbf{R}_n \right|^2 \cdot \sin \phi \cdot \Delta \theta \cdot \Delta \phi,
\]

where \( \sin \phi \cdot \Delta \theta \) and \( \Delta \phi \) are angles between the lines which connect \( \mathbf{R}_n \) to the edges of the unit area (Fig. 1b).

The total travel distance of this particle is represented by \( D \):

\[
D = \left| \mathbf{r}_1 \right| + \left| \mathbf{r}_2 \right| + \left| \mathbf{r}_3 \right| + \cdots + \left| \mathbf{r}_n \right| + \left| \mathbf{R}_r - \mathbf{R}_n \right|,
\]

and the travel time from \( \mathbf{R}_o \) to \( \mathbf{R}_r \) is given by \( D/v \). We divide the lapse time \( t \) into segments of width \( \Delta t \), as shown in Fig. 2, where the lapse time \( t \) is measured from the origin time. The first particle with the probability given by (3) falls into a time interval segment which includes the travel time (Fig. 2a). The second particle shot from the source (\( \mathbf{R}_o \)) takes the same procedure (Fig. 2b). The travel distance of the \( m \)-th particle is represented by

\[
D_m = \left| \mathbf{r}_{1,m} \right| + \left| \mathbf{r}_{2,m} \right| + \left| \mathbf{r}_{3,m} \right| + \cdots + \left| \mathbf{r}_{n,m} \right| + \left| \mathbf{R}_r - \mathbf{R}_{n,m} \right|
\]

(4)
The subscript \( m \) representing the sequence of the particle goes from one to \( W_o \), where \( W_o \) is the total number of particles. The probability given by (3) for the \( m \)-th particle is added to the number stored in the time interval segment corresponding to the travel time \( D_m/v \) (Fig. 2c). The black bar height in Fig. 2 shows the newly added number for the \( m \)-th particle in terms of a probability.

Let each particle have unit energy; hereafter the total number of the particles, \( W_o \), represents the energy radiated from the source. After the calculation of a sufficiently large number of shot particles, dividing the value stored in each time interval segment by the wave velocity \( v \) and the width of the time interval segment \( \Delta t \), we obtain the energy density function due to the \( n \)-th order scattered energy particles with shot energy \( W_o \). This density function is considered to be equivalent to that of
Fig. 2  Procedure to obtain the energy density of the $n$-th scattered wave from a sufficiently large number of particles. (a) The first particle falls into a time interval segment with the probability given by (3), (b) the second particle and (c) the $m$-th particle. $D_m$ is the total travel distance of the $m$-th particle, $D_m = |\mathbf{r}_{1,m}| + |\mathbf{r}_{2,m}| + |\mathbf{r}_{3,m}| + \cdots + |\mathbf{r}_{n,m}| + |\mathbf{R}_n - R_{m,m}|$ where the second subscript of each $\mathbf{r}_n$ and $\mathbf{R}_n$ denotes the sequence of the particle. The black bar height shows the newly added number by the particle in terms of a probability.
### Table 1 List of parameters used in the simulation.

<table>
<thead>
<tr>
<th>Scattering coefficient $\bar{g}$ (km$^{-1}$)</th>
<th>Maximum order of scattering considered in the simulation</th>
<th>Number of particles $W_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>20</td>
<td>$5 \times 10^5$</td>
</tr>
<tr>
<td>0.02</td>
<td>20</td>
<td>$5 \times 10^6$</td>
</tr>
<tr>
<td>0.05</td>
<td>20</td>
<td>$5 \times 10^6$</td>
</tr>
<tr>
<td>0.1</td>
<td>20</td>
<td>$5 \times 10^6$</td>
</tr>
<tr>
<td>0.2</td>
<td>100</td>
<td>$2 \times 10^6$</td>
</tr>
<tr>
<td>0.5</td>
<td>100</td>
<td>$1 \times 10^6$</td>
</tr>
</tbody>
</table>

the energy density function of the coda wave due to the $n$-th order scattered waves.

Summing up the energy density functions of scattering up to a sufficiently large order, we can obtain the energy density function at the hypocentral distance, $|\mathbf{R}_s - \mathbf{R}_p|$, including the multiple scattering effect. Varying the hypocentral distance, we can evaluate the spatial change in the energy density of multiple scattered waves.

In this paper we make six simulations using the parameters listed in Table 1 for a case equivalent to $\Delta t = 0.0292$ s, $v = 4.0$ km/s. In each simulation the range of lapse times ranges from $0$ s to $26.25$ s and the interval between receivers is $1$ km in a range of $0 \leq r \leq 100$ km, where $r$ is the distance between the source and the receiver. Hereafter values used in the simulations are represented by numerals with overbar $\bar{r}$, that is $\bar{g}$, $\bar{T}$, $\bar{r}$ and $\bar{v}$, to avoid the confusion between these values and the actually observed ones. The energy density is obtained in ranges of non-dimensional time $\bar{g} \bar{v} \bar{T} \leq 52.5$ and non-dimensional hypocentral distance $\bar{g} \bar{r} \leq 50$. We may consider that the values of $\bar{g} \bar{v} \bar{T} = 52.5$ and $\bar{g} \bar{r} = 50$ are large enough to apply these simulations to actually observed data. Accuracy of our simulations was discussed in detail in a previous paper (Hoshiba, 1991).

### 3. Method for separating $Q_{e1}$ and $Q_{i1}$ from $Q_{e1}$

In section 2, we simulated energy den-

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**Fig. 3** Schematic procedures to estimate $4 \pi r^2 \cdot E_1(r)$, $4 \pi r^2 \cdot E_2(r)$ and $4 \pi r^2 \cdot E_3(r)$.

(a) $E_i(r)$ represents the energy of direct wave and some scattered wave until $r/v + T_i$, $E_1(r)$ is that of scattered wave from $r/v + T_1$ to $r/v + T_2$ and $E_i(r)$ is from $r/v + T_2$ to $r/v + T_3$ at the hypocentral distance, $r$.

(b) Schematic plots of $4 \pi r^2 \cdot E_1(r)$, $4 \pi r^2 \cdot E_2(r)$ and $4 \pi r^2 \cdot E_3(r)$ against the hypocentral distance, $r$. 
sity for case of $Q_l^{-1} = 0$, but hereafter the effect of intrinsic absorption is taken into account. We now discuss how the calculation described in the previous section can be applied to determine $Q_s^{-1}$ and $Q_l^{-1}$ from observations.

Let $E(r, t | \omega)$ be the energy density for both direct and scattered waves at hypocentral distance, $r$, and lapse time $t$ from the origin time for the frequency band around central angular frequency $\omega$. We do not consider the wave conversion between different frequencies as listed in section 2, so the estimation of $Q_l^{-1}$ and $Q_s^{-1}$ can be done in each angular frequency band independently. Omitting the symbol $\omega$, we write $E(r, t)$ in representing $E(r, t | \omega)$ in the following. Here, let $E_1$, $E_2$ and $E_3$ be integrals for three different lapse time windows,

$$E_1(r) \equiv v \cdot \int_{r/v}^{r/v + T_1} E(r, t) dt$$
$$E_2(r) \equiv v \cdot \int_{r/v + T_1}^{r/v + T_2} E(r, t) dt$$
$$E_3(r) \equiv v \cdot \int_{r/v + T_2}^{r/v + T_3} E(r, t) dt.$$  \hspace{1cm} (5)

Fixed time $T_1$, $T_2$ and $T_3$ for the window of the integrals are measured from the S wave onset $r/v$, not from the origin time as schematically shown in Fig. 3a. $E_1(r)$ represents the integrated energy of the direct wave and earlier coda waves, $E_2(r)$ stands for that of a later part of coda waves. $E_3(r)$ is that of a middle portion (Fig. 3a). $E_2$ and $E_3$ are composed of scattered waves only. Instead of $E_1(r)$, $E_2(r)$ and $E_3(r)$, we plot $4\pi r^2 \cdot E_1(r)$, $4\pi r^2 \cdot E_2(r)$ and $4\pi r^2 \cdot E_3(r)$, against hypocentral distance $r$, as shown in Fig. 3b.

Here the technique to obtain $4\pi r^2 \cdot E_1(r)$, $4\pi r^2 \cdot E_2(r)$ and $4\pi r^2 \cdot E_3(r)$ for unit source power from the simulation is described. We first carry out integrals (5) for parameters $\bar{F}$ and $\bar{V}$, and then re-scale them to $t$ and $r$. Therefore the ranges of $\bar{F}$ and $\bar{V}$ are different from those of $t$ and $r$. Let $L_s$ be defined as

$$1/L_s \equiv (Q_s^{-1} + Q_l^{-1}) \cdot \omega/v,$$  \hspace{1cm} (6)

which is called extinction length in Wu and Aki (1988). The seismic wave is composed of two parts: direct wave and scattered waves. Let $E_s(\bar{F}, \bar{V})$ be the energy density of scattered waves obtained by the method described in section 2, in which $g = \bar{g}$ and $v = \bar{v}$ are used. When $L_e$ and $B_0 \equiv Q_s^{-1}/Q_l^{-1}$ are given, multiplying direct wave energy by the total attenuation factor and scattered wave energy by the intrinsic absorption factor, we obtain $4\pi r^2 \cdot E_1(r)$, $4\pi r^2 \cdot E_2(r)$ and $4\pi r^2 \cdot E_3(r)$ from $E_s(\bar{F}, \bar{V})$,

$$4\pi r^2 \cdot E_1(r) = \exp(-L_e^{-1} \cdot \bar{F}) + 4\pi r^2 \cdot \frac{v}{W_0} \cdot \int_{\bar{F}/\bar{V}}^{\bar{F}/\bar{V} + T_1} E_s(\bar{F}, \bar{V}) \cdot \exp(-h \cdot \bar{V}/\bar{F}) d\bar{V}$$
$$4\pi r^2 \cdot E_2(r) = 4\pi r^2 \cdot \frac{v}{W_0} \cdot \int_{\bar{F}/\bar{V} + T_1}^{\bar{F}/\bar{V} + T_2} E_s(\bar{F}, \bar{V}) \cdot \exp(-h \cdot \bar{V}/\bar{F}) d\bar{V}$$
$$4\pi r^2 \cdot E_3(r) = 4\pi r^2 \cdot \frac{v}{W_0} \cdot \int_{\bar{F}/\bar{V} + T_2}^{\bar{F}/\bar{V} + T_3} E_s(\bar{F}, \bar{V}) \cdot \exp(-h \cdot \bar{V}/\bar{F}) d\bar{V}$$  \hspace{1cm} (7)

where

$$L_e^{-1} = \bar{g} \cdot (1/B_0),$$
$$h = L_e^{-1} \cdot (1 - B_0) = \bar{g} \cdot (1 - B_0)/B_0,$$
$$T_1 = |B_0 \cdot v/(L_e \cdot \bar{g} \cdot \bar{v})| \cdot T_1,$$
$$T_2 = |B_0 \cdot v/(L_e \cdot \bar{g} \cdot \bar{v})| \cdot T_2,$$
$$T_3 = |B_0 \cdot v/(L_e \cdot \bar{g} \cdot \bar{v})| \cdot T_3,$$
$$\bar{F} = |B_0 \cdot v/(L_e \cdot \bar{g})| \cdot \bar{r}.$$
The first term on the right side of $E_1(r)$ represents the energy of the direct wave decreased by total attenuation. The exponential factor, $\exp \left( -\frac{h}{\nu} \tilde{T} \right)$, in each integral means the intrinsic absorption which was removed in the simulation described in section 2. In this paper, we choose $T_1 = 15s$, $T_2 = 30s$ and $T_3 = 45s$ and assume $\nu = 4.0$ km/s as $S$ wave velocity in the crust and upper mantle. Fig. 4 shows the energy distributions against the hypocentral distance $r$ (km) in a range of $r \leq r_{\text{max}} = 250$ km for different pairs of $L_e$ and $B_o$. Different line thickness is used to distinguish different time windows, different line type is used for different value of $B_o$. The curves are given by $B_2(\tilde{T}, \tilde{T})$ corresponding to the smallest $\tilde{g}$ of six cases ($\tilde{g} = 0.01, 0.02, 0.05, 0.1, 0.2, 0.5$ km$^{-1}$) which satisfies

$$\tilde{g} \nu \tilde{T} \text{max} \geq \frac{B_0}{L_e} \cdot \nu \cdot (T_3 + \frac{r_{\text{max}}}{\nu}),$$

(8)

where $\tilde{T}_{\text{max}}$ is the maximum lapse time in the simulation (in this paper $\tilde{T}_{\text{max}} = 26.25$ s). In Fig. 4 we show three curves for each seismic albedo that are generated using the $\tilde{g}$ that gives the finest sampling of the energy integrals as hypocentral distance as described by the condition in Eq. (8).

In observational analysis, $E_1, E_2$ and $E_3$ can be estimated from the band-pass filtered seismogram's power. In practice, we compare observed $4\pi r^2 \cdot E_1(r)$, $4\pi r^2 \cdot E_2(r)$ and $4\pi r^2 \cdot E_3(r)$ with numerically simulated ones shown in Fig. 4, and then we can choose the most appropriate pair of $L_e$ and $B_0$. $Q_s^{-1}$ and $Q_l^{-1}$ are given by the following relations from the estimated $L_e$ and $B_0$:

$$Q_s^{-1} = \frac{\nu \cdot B_0}{(L_e \cdot \omega)},$$

$$Q_l^{-1} = \nu \cdot (1 - B_0)/(L_e \cdot \omega).$$

We show three curves, $4\pi r^2 \cdot E_1(r)$, $4\pi r^2 \cdot E_2(r)$ and $4\pi r^2 \cdot E_3(r)$, for two $L_e^{-1}$ values ($0.01$ and $0.04$ km$^{-1}$) and two $B_0$ values ($0.2$ and $0.8$) in Fig. 5. We can see from this figure that the decay rate of $4\pi r^2 \cdot E_i(r)$ with distance is mainly affected by $L_e^{-1}$, and the ratio of $4\pi r^2 \cdot E_i(r)/4\pi r^2 \cdot E_2(r)$ or $4\pi r^2 \cdot E_1(r)/4\pi r^2 \cdot E_3(r)$ is mainly affected by $B_0$.

Because the direct wave energy changes rapidly into the scattered wave energy in a medium of large $g$ ($=L_e^{-1} \cdot B_0$), $E_2(r)$ or $E_3(r)$ is larger than $E_1(r)$ in such a medium for a range of large hypocentral distance.

4. Spatial distribution of energy density of coda waves

The energy integrals in Fig. 4 are those of the case where the source radiates unit energy and all site amplification factors are the same. Therefore, before applying the method described in the previous section, it is necessary to correct different source powers and different site amplifications.

For the correction, the assumption of the coda wave method has often been used (e.g. Aki, 1969; Aki and Chouet, 1975; Aki, 1980; Phillips and Aki, 1986); coda wave energy is distributed nearly uniform within a spherical volume far behind the S wave front. Unfortunately, however, the validity of the assumption has hardly been examined. In this section, we will examine the spatially uniform distribution of energy density of coda waves.

The parameter which controls the spatial distribution of energy density of coda wave is $g = L_e^{-1} \cdot B_0$. Fig. 6 shows the distributions at $t = t_{\text{ref}}$ ($= 30s, 45s, 60s$ and $100s$) measured from the origin time, which is called reference time, for $g = 0.005, 0.010, 0.030$ and $0.050$ km$^{-1}$ estimated from the numerical simulation. The reference times have often been chosen on the condition that $t_{\text{ref}}$ equals $2\cdot r/\nu$ or is larger. Then we show the range of $r$ which satisfies $t_{\text{ref}} \geq 2\cdot r/\nu$. The assumption of the uniform distribution in the range of the hypocentral distance should be examined.

From Fig. 6, for the medium of small $g$ the distributions at all $t_{\text{ref}}$ are nearly uniform, so the assumption of uniform distribution holds good in such a medium. But for the medium of large $g$, especially at $t_{\text{ref}} =$
Fig. 4 Energy distribution curves of $4\pi r^2 E(r)$, $4\pi r^2 E_s(r)$, and $4\pi r^2 E_x(r)$ as a function of hypocentral distance $r$ (km) for different pairs of $L_0$ and $B_0$ predicted from the simulations, in which $T_1$, $T_2$, and $T_3$ in (b) and (c) are chosen to be 15, 30, and 45 s respectively. Different line thicknesses are used to distinguish $E_r$, $E_s$, and $E_x$, different line types are used for different values of $B_0$. 

$L_0 = 0.0030 / km$
Fig. 4 Continued.
Fig. 4 Continued.
Fig. 4 Continued.
Fig. 4 Continued.
Fig. 4 Continued.
100s, the distributions are not uniform, so the assumption does not hold good in such a strong scattering medium.

After estimating $L_x^{-1}$ and $B_0$ by our method from data in which different source powers and different site amplifications are corrected by the coda wave method, we have to examine the assumption of uniform distribution by using Fig. 6. Even if the assumption holds good, it is not assured that there are no errors in estimated $L_x^{-1}$ and $B_0$ because the data are analyzed based on the assumption. Therefore, it is necessary to confirm the self-consistency between the assumption and the estimated values. If the assumption does not hold, there is a discrepancy definitely between assumption and results; the estimated $L_x^{-1}$ and $B_0$ must contain some errors.
5. Example and discussion

Here we briefly show an example of application of the method described in section 3; detailed discussion will be given by Fehler et al. (1991) who estimated \(Q_{e}^{-1}\) and \(Q_{t}^{-1}\) separately for the Kanto and Tokai region in Japan. However, they took 15 s, 30 s and 100 s for \(T_1\), \(T_2\) and \(T_3\) respectively (Note that \(T_3\) in Fig. 4 is 45 s). In Fig. 7a, the estimated energy distributions are shown for 2-4 Hz band-pass filtered vertical component energy after corrections for different source powers and different site amplifications. The running averages of 
\[
\log_{10} \left[ 4 \pi r^2 \cdot E_i(r) \right], \quad \log_{10} \left[ 4 \pi r^2 \cdot E_s(r) \right]
\]
and 
\[
\log_{10} \left[ 4 \pi r^2 \cdot E_3(r) \right]
\]
on 15 km windows with 7.5 km spacing are also shown. The source power and the site amplification factor were corrected by using the coda wave method of Phillips and Aki (1986) described in section 4. To estimate the site amplification, they compared the amplitude of the coda wave with the signal amplitude at a reference station measured at the same lapse time of the same earthquake. To estimate the source power they measured the coda amplitude at some constant lapse time after the earthquake origin time at one station. Source power estimates made using site-corrected data from many stations were averaged to obtain more reliable results. It is possible relatively but impossible absolutely to estimate the source power and site amplification factor by these corrections. Therefore they pay their attention only to the shapes of curves and ratios of 
\[
4 \pi r^2 \cdot E_i(r) / 4 \pi r^2 \cdot E_s(r) \quad \text{or} \quad 4 \pi r^2 \cdot E_1(r) / 4 \pi r^2 \cdot E_3(r)
\]
in Fig. 4, not to the absolute levels of energy integrals, when they estimated the most appropriate pair of \(L_e^{-1}\) and \(B_0\) from an equivalent figure to Fig. 4. By comparing the energy integrals with the curves predicted from the simulation, they estimated that \(1/L_e = 0.019 \text{ km}^{-1} (\pm 0.007 \text{ km}^{-1})\) and \(B_0 = 0.34 (\pm 0.06)\); the most appropriate curves are shown in Fig. 7b. Then, they estimated that \(Q_{e}^{-1} = 1.4 \times 10^3\) and \(Q_{t}^{-1} = 2.6 \times 10^3\), when \(v = 0.4 \text{ km/s}\).
Fig. 7a Examples of energy distribution as a function of hypocentral distance \( r(\text{km}) \) (after Fehler et al., 1991), where \( T_s \) was chosen to be 100 s. \( 4\pi r^2 \cdot E_s(r) (\bigcirc) \), \( 4\pi r^2 \cdot E_c(r) (\bigotimes) \) and \( 4\pi r^2 \cdot E_z(r) (\times) \) are observed energy distributions of 2-4 Hz band pass filtered vertical component at Kanto and Tokai regions in Japan after corrections of source powers and site amplifications. TRU is the name of the reference station. The averages of \( \log_{10} [4\pi r^2 \cdot E_s(r)] \), \( \log_{10} [4\pi r^2 \cdot E_c(r)] \) and \( \log_{10} [4\pi r^2 \cdot E_z(r)] \) over 15 km windows with 7.5 km spacing are shown by the curves.

is assumed.

Here we examine the uniform distribution described in section 4. Fehler et al. (1991) chose around 60 s as \( t_{ref} \), and \( g \left( = L_{s}^{-1} \cdot B_{o} \right) \) was here estimated to be 0.007 km\(^{-1}\). From Fig. 6, we can see that the distribution of coda wave energy of such a case is considered to be nearly uniform. This shows the self-consistency between the assumption and the estimated \( L_{s}^{-1} \) and \( B_{o} \).

The method for the separation of \( Q_{s}^{-1} \) and \( Q_{i}^{-1} \) described in this paper is based on simple assumptions listed in section 2. A question remains about the validity of the simple assumptions in the application of the model to the real earth, especially of isotopic scattering and uniform structure of \( Q_{s}^{-1} \) and \( Q_{i}^{-1} \). Though there are some scatter in data from the averages in Fig. 7a, the fitness of the simple model to the data seems to be good as shown in Fig. 7b. If the observed energy distributions did not fit any case of Fig. 4, this must be due to the difference of our assumptions from the properties of the real earth. Anisotropic scattering and non-uniform scattering strength and non-uniform absorption may change the arch shapes and move the
curves up and down in Fig. 4.

In general, forward scattering increases with increasing frequency (Sato, 1984; Matsumoto and Hasegawa, 1989). If we adopt large forward scattering in our model, the energy integral, $E_i(r)$, will increase relative to $E_s(r)$ and $E_0(r)$ when the energy integrals are compared to results of the isotropic case. The effect of anisotropic scattering was explained qualitatively by Gusev and Abubakirov (1987). If we use Fig. 4 to fit data for such an anisotropic scattering case, we will obtain a seismic albedo smaller than 1 even if there is no intrinsic absorption. Thus we should remember that seismic albedos estimated from Fig. 4 may reflect the anisotropy in scattering.

The values of $g$ and $Q_i^{-1}$ may decrease with depth in the earth. Because $E_2(r)$ and $E_3(r)$ are affected by the properties of the deeper structure more than $E_i(r)$, the decrease of $g$ moves $E_2(r)$ and $E_3(r)$ curves down in comparison with $E_i(r)$ and the decrease of $Q_i^{-1}$ moves them up. The non-uniform structures of $g$ and $Q_i^{-1}$ also have an effect on the determination of $L_e^{-1}$ and $B_0$.

Thus we have to remember the basic assumptions of our model, especially the spatially uniform $g$ and $Q_i^{-1}$, and isotropic scattering, when we interpret the meanings of $L_e^{-1}$ and $B_0$ by our method; these assumptions are mathematically simple, but have not been confirmed by the observation measurements.

Wu (1985) previously proposed a method to separate scattering attenuation and intrinsic absorption from total attenuation based on an energy integral over one infinite time window, but it was difficult, in practice, to apply Wu's method to observations. Here we propose a method based on energy integrals of multiple time windows. At least two time windows are necessary for the analysis, but we note that three are very useful. We can analyze $E_2$ and $E_3$, even when direct waves are clipped and/or $E_1$ and $E_2$ for seismograms in which the latter part of coda is contaminated by strong
noise. We call our method "multiple lapse time window analysis" in comparison with Wu's (1985) single lapse time window analysis.

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地震波形全体の解析による散乱減衰と吸収減衰の分離
——多重散乱波生成のシミュレーションをもとにして——

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地震波の減衰には 2 つの要因、散乱と吸収があると考えられている。吸収では波動のエネルギーが熱へと変換し失われるのでに対し、散乱は直達波の波動エネルギーを減少させるものの散乱波を励起するために、全エネルギーを減らさないとはいない。ここでは、散乱の等方性および散乱体・吸収体の一様構造の仮定のもとで、この 2 つの要因を分離する新しい方法を提案する。この方法は地震波形の 3 つの部分、すなわち、直達波と earlier part の散乱波の部分、middle part の部分、later part の部分、に分け、それぞれの部分の地震波エネルギーの時間積分を震源距離の関数として表したものと、モンテカルロ法を用いたシミュレーション結果の時間積分とを比較することにもとづいている。具体的には、おのおのおの時間積分の値の大小は seismic albedo, B₀（＝散乱減衰量／全減衰量）に依存しているので、全減衰量と B₀ の多くのペアに対して、シミュレーション結果の時間積分を計算しておき、実際の観測波形の時間積分と比較することにより、2 つの要因の分離を行うという方法である。