Multiple isotropic scattering model including $P$–$S$ conversions for the seismogram envelope formation

Haruo Sato

Geophysical Institute, Faculty of Sciences, Tohoku University, Aoba-ku, Sendai-shi 980, Japan

Accepted 1993 October 26. Received 1993 October 26; in original form 1993 March 9

SUMMARY
High-frequency seismogram envelopes of local earthquakes are well characterized by the scattering nature of the randomly inhomogeneous lithosphere. In addition to direct $P$- and $S$-wave phases, incoherent wave trains are found not only in the $S$ coda but also between the $P$ and $S$ phases. This implies the importance of scattering with conversions between $P$ and $S$. The contribution from scattering increases with increasing hypocentral distance or lapse time because of multiple scattering processes. The multiple scattering contribution has been extensively studied mathematically especially for the $S$ coda by supposing isotropic scattering. Here, extending the isotropic scattering assumption for scattering in conversion between two different wave velocities, we formulate the multiple scattering process for spherical radiation from a point source and derive the space time distribution of the energy density in 3-D space on the basis of energy transport theory. We confirm conservation of the total energy in the case of no intrinsic absorption, which assures the self-consistency of the formulation. A hybrid method, combining the analytical solutions corresponding to single scattering and numerical calculations for multiple scattering with order greater than, or equal to, two, is proposed to synthesize the spacetime distribution of the energy density. Synthesized time traces give a good explanation of the rather smooth and gradual increase in amplitude of wave trains often observed between $P$ and $S$ phases and the larger amplitude of the $S$ coda. The envelope characteristics, except for short time windows around the direct wave arrivals, are well characterized by multiple scattering irrespective of the change in source duration time.

Key words: $P$ waves, scattering, seismic coda, $S$ waves.

1 INTRODUCTION

Envelopes of high-frequency seismograms, especially $S$-coda waves, of local earthquakes have been studied on the basis of scattering in the randomly inhomogeneous lithosphere (Aki & Chouet 1975). $S$ coda has been studied and analysed on the basis of $S$ to $S$ scattering, since it is interpreted to be mostly composed of scattered $S$ waves (Hoshiba 1991; Mayeda et al. 1991; Fehler et al. 1992). Frankel & Wennerberg (1987) proposed a model incorporating large forward scattering derived from numerical calculations with energy conservation. Recent observations show that envelope characteristics, such as peak delay and the broadening of $S$ waves are very useful to classify the regional differences of randomness on the basis of multiple $S$ to $S$ forward scattering or strong diffraction of $S$ waves (Gusev & Abubakirov 1987; Sato 1989; Scherbaum & Sato 1991; Obara & Sato 1993). We often observe a rather smooth and gradual increase in amplitude of wave trains between $P$ and $S$ phases and the larger amplitude of $S$ coda. We occasionally observe a seismogram of which the envelope looks like a spindle starting from the $P$-wave onset without a clear direct $S$ phase in volcanic areas or at the back-arc side of the volcanic front in a subduction zone (Obara & Sato 1993). These phenomena indicate the importance of $P$–$S$ conversion in scattering in the formation of the high-frequency seismogram envelopes. The importance of $P$–$S$ conversion scattering is theoretically shown in scattering amplitudes of localized elastic inhomogeneities (Sato 1984; Wu & Aki 1985).

There are several ways to synthesize seismogram envelopes in inhomogeneous media. One is exact, based on the elastic wave equation. Sato (1984) proposed a way to synthesize three-component seismogram envelopes by
summing up all four conversion scattering amplitudes from distributed randomness on the basis of the Born approximation, where only single scattering was considered. The parabolic approximation is useful for the envelope synthesis when the spatial scale of randomness is longer than the wavelength of seismic waves studied (Sato 1989). But, when the spatial spectra of velocity inhomogeneities are rather broad, we will have to consider P-S conversion scattering seriously. There have been some attempts to synthesize, not just the envelope, but the seismogram itself as in Fisk, Charrette & McCartor (1992) and Wu (1993), who focus on forward scattering adopting a phase screen method, in which backward scattering is discarded. Benites, Aki & Yomogida (1992) synthesized an SH seismogram of multiple scattering due to 50 circular cavities based on a boundary integral method. These approaches are rigorous but need tremendous numerical calculations.

Another approach is based on energy transport theory, which simply supposes isotropic scattering due to point-like scatterers randomly distributed and a geometrical spreading factor determined by the topographic dimension of space. The single scattering case was formulated and solved by Sato (1977a). Wu (1985) tried to extend it to multiple scattering for the stationary case. As rigorously formulated by Shang & Gao (1988) in the 2-D case, this approach makes it possible to synthesize the envelope including all orders of multiple scattering. The extension to the 3-D case was done by Zeng, Su & Aki (1991), and the derivation of the analytical solution was studied by Sato (1993a) for 1- and 2-D cases. The simplest way to incorporate P-S conversion scattering with this scheme is to assume isotropic scattering for the conversion scattering. The scattering pattern will be non-isotropic at high frequencies, as can be shown from the Born approximation (Sato 1984; Wu & Aki 1985); however, the isotropic assumption simplifies the mathematics and the synthesized envelope is valuable as a reference for the study of the non-isotropic case. Furthermore, this formalism includes another important contribution to P-S conversion scattering: a large change in propagation speed. This extension was first made by Sato (1977b) for the case of single scattering.

Here, a mathematical formulation of a multiple isotropic scattering process including P-S conversion for the seismogram envelope synthesis is presented on the basis of energy transport theory. Recently, Zeng (1993) presented a similar formulation, in which a numerical integration method to assure the convergence was proposed. We here focus on energy conversion which assures the mathematical consistency and the derivation of analytical solutions corresponding to single scattering. The subtraction of the single scattering term enables us to calculate easily the numerical integraton of the multiple scattering term. Numerically synthesizing the energy density for two sets of scattering parameters, we examine the characteristics of the time trace for the energy density.

2 MATHEMATICAL FRAMEWORK

Here, we suppose the impulsive spherical radiation of energy of P and S waves, $W_P$ and $W_S$, from the hypocentre at the origin and the isotropic scattering for four scattering modes: P to P, P to S, S to P and S to S scattering. The different modes are characterized by scattering parameters, $\zeta_{PP}$, $\zeta_{PS}$, $\zeta_{SP}$ and $\zeta_{SS}$ in the two integral equations:

$$
E_P(r, t) = W_P G_P(r, t) + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \zeta_{PP} E_P(t', t') G_P(t - t', t - t') \right] dt' \ dr'
$$

$$
E_S(r, t) = W_S G_S(r, t) + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \zeta_{SS} E_S(t', t) \right] G_S(t - t', t - t) dt' \ dr'
$$

where $E_P$ and $E_S$ are the energy densities for P and S waves. The energy density $E$ is written as a sum:

$$
E(r, t) = E_P(r, t) + E_S(r, t).
$$

We suppose the Green's functions represent geometric spreading in 3-D space and include causality:

$$
G_P(r, t) = \frac{1}{4\pi r^2} \delta \left( t - \frac{r}{\alpha} \right) \exp \left[ -\left( \zeta_{PP} + \zeta_{PS} + \zeta_{SP} \right) t \right]
$$

for $t \geq 0 = 0$ for $t < 0$

$$
G_S(r, t) = \frac{1}{4\pi r^2} \delta \left( t - \frac{r}{\beta} \right) \exp \left[ -\left( \zeta_{SS} + \zeta_{SP} + \zeta_{SS} \right) t \right]
$$

for $t \geq 0 = 0$ for $t < 0$

where $\alpha$ and $\beta$ represent the P- and S-wave velocities, and $\gamma = \alpha/\beta > 1$. $\zeta_P$ and $\zeta_S$ are the parameters for the intrinsic absorption. The scattering system is assumed to be exactly described by the above three eqs. (1)–(3), not only in the far field but also in near field since scatterers are assumed to be point-like. Scattering increases the coda portion of energy densities; however, intrinsic absorption attenuates the total energy. This system has no specific orientation, therefore, we may write the argument of the energy densities as $r = |r|$ instead of $r$ in the following.

We are going to solve eqs (1)–(3) by using the Fourier transform in space and the Laplace transform in time. We here define the transformation of the Green's functions (3) as

$$
G_P(k, s) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \int_{0}^{\infty} \exp (-ikr) \right] G_P(r, t) \ dt \ dr
$$

$$
G_S(k, s) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ \int_{0}^{\infty} \exp (-ikr) \right] G_S(r, t) \ dt \ dr
$$

where the argument is $k = |k|$ because there is no specific orientation. Taking the Fourier transform in space and the Laplace transform in time of the convolution integrals, eq.

$$
= \frac{1}{\alpha k} \tan^{-1} \left( \frac{ak}{s + \zeta_{PP} + \zeta_{PS} + \zeta_{SP}} \right)
$$

$$
= \frac{1}{\beta k} \tan^{-1} \left( \frac{\beta k}{s + \zeta_{SS} + \zeta_{SP} + \zeta_{SS}} \right)
$$
First, we examine the temporal change in the total energy. We can easily evaluate the total energy defined as a sum of spatial integrals of $E_p$ and $E_s$, which is given by the Fourier transformation at wavenumber $k = 0$. For the $P$ wave, we have

$$
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dt E_P(r, t) = E_P(k = 0, s) = \frac{W_p + \mu_P}{(s + a)^2 - b^2}, \quad (6)
$$

where

$$
a = \frac{1}{2}(\xi_{ps} + \xi_{sp} + \xi_P + \xi_S),
$$

$$
b = \frac{1}{2} \sqrt{(\xi_{ps} + \xi_P - \xi_{sp} - \xi_S)^2 + 4\xi_{ps} \xi_{sp}},
$$

$$
\mu_P = (W_p + W_s) \xi_{sp} + W_p \xi_S.
$$

The inverse Laplace transformation of eq. (6) to the time domain is given by

$$
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dt E_P(r, t) = E_P(k = 0, t) = \frac{\mu_P - W_P(a - b)}{2b} \times \exp[-(a - b)t] + \frac{W_P(a + b) - \mu_P}{2b} \times \exp[-(a + b)t]. \quad (8)
$$

For $S$ waves, we get

$$
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dt E_S(r, t) = E_S(k = 0, t) = \frac{\mu_S - W_S(a - b)}{2b} \times \exp[-(a - b)t] + \frac{W_S(a + b) - \mu_S}{2b} \times \exp[-(a + b)t]. \quad (9)
$$

where

$$
\mu_S = (W_p + W_s) \xi_{ps} + W_s \xi_S.
$$

For a large lapse time, the first term is dominant in each of eq. (8) and eq. (9) since the second term rapidly decreases because $a \gg b > 0$.

When the intrinsic absorption is the same for $P$ and $S$ waves,

$$
\xi_P = \xi_S = \xi'.
$$

we can simplify eqs (8) and (9) as

$$
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dt E_P(r, t) = \frac{\xi_{sp}(W_p + W_s)}{\xi_{ps} + \xi_{ps}} \exp(-\xi't) + \frac{W_p \xi_{ps} - W_s \xi_{ps}}{\xi_{sp} + \xi_{ps}} \exp[-(\xi_{ps} + \xi_{sp} + \xi')t]
$$

$$
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dt E_S(r, t) = \frac{\xi_{sp}(W_p + W_s)}{\xi_{sp} + \xi_{ps}} \exp(-\xi't) + \frac{W_S \xi_{sp} - W_p \xi_{ps}}{\xi_{sp} + \xi_{ps}} \exp[-(\xi_{ps} + \xi_{sp} + \xi')t] \quad (10)
$$

The partition of energy into $P$- and $S$-wave modes for a large lapse time is controlled by the ratio of conversion scattering parameters in the first term of each equation. The last equation clearly displays the meaning of the intrinsic absorption parameter $\xi'$. Thus, the total energy is conserved in the case of no intrinsic absorption. This demonstrates the mathematical self-consistency of the set of eqs (1)–(3). We schematically illustrate the temporal change in $P$- and $S$-wave energy, and the conservation of total energy in Fig. 1 for the case of no intrinsic absorption.

3 ANALYTICAL DERIVATION OF THE SINGLE SCATTERING TERM

By using the inverse Fourier transformation in space and the inverse Laplace transformation in time of eq. (5), we can formally solve the space–time distribution of energy density. But, the convergence of the numerical integration is slow since the integral kernel rapidly oscillates for large wavenumbers. Zeng (1993) recently presented a way to calculate these integrals numerically. The convergence problem comes from two singular terms: the direct energy density given by a delta function and the single scattering energy density which logarithmically diverges at the wave front in the time domain (Sato 1977a, b). Therefore, we
decompose eq. (5) into three terms: the direct energy density \( E^0 \) and the single scattered energy density \( E^1 \), and the energy density of multiple scattering of order greater than, or equal to, two, \( E^m \) as follows:

\[
E(k, s) = E^0(k, s) + E^1(k, s) + E^m(k, s), \tag{13}
\]

\[
E^0(k, s) = W_P G_p + W_S G_s
\]

\[
E^1(k, s) = W_P G_p (\xi_{pp} G_p + \xi_{ps} G_s) + W_S G_s (\xi_{ps} G_p + \xi_{ss} G_s) \tag{14}
\]

and

\[
E^m(k, s) = \frac{1}{(1 - \xi_{pp} G_p)(1 - \xi_{ss} G_s) - \xi_{ps} \xi_{sp} G_p G_s} \times \left[ W_P G_p (\xi_{ps} G_s (1 - \xi_{pp} G_p) + \xi_{sp} G_p (1 + \xi_{ps} G_s)) + \xi_{pp} G_p (\xi_{pp} G_p (1 - \xi_{ss} G_s) + \xi_{sp} G_p (1 + \xi_{sp} G_s)) \right] + W_S G_s (\xi_{sp} G_s (1 - \xi_{ps} G_p) + \xi_{ss} G_p (1 + \xi_{ps} G_s)) \right]. \tag{15}
\]

When the total attenuation as the sum of scattering attenuation and intrinsic absorption is the same for \( P \) and \( S \) waves,

\[
\xi_{pp} + \xi_{ps} + \xi_{sp} = \xi_{ss} + \xi_{sp} + \xi_{sp} = \eta. \tag{16}
\]

There have been few measurements of these parameters except \( \xi_{ss} \): Sato (1990) estimated \( \xi_{ss} \approx 30 \) s from the measurement of coda excitation in Kantou, Japan. The ratio of attenuation observed indicates the above ratio:

\[
Q_p^{-1} = (\xi_{pp} + \xi_{ps} + \xi_{sp}) \quad Q_s^{-1} = (\xi_{ss} + \xi_{sp} + \xi_{sp}) = 0.7 - 2
\]

in the lithosphere for frequencies higher than 1 Hz (see Yoshimoto, Sato & Ohtake 1993, Fig. 7). Therefore, the above choice is reasonable. We can solve \( E^1 \) analytically in space and time when eq. (16) is valid. The inverse Fourier transformation is written as an integral along the real \( k \) axis, since the integral is not a function of vector \( k \) but of the magnitude \( k = |k| \) only. For example, the energy density due to single \( PS \) scattering is

\[
E_{PS}(r, s) = \frac{W_P W_{PS}}{4 \pi \alpha r} \int \int G_p(k, s) G_s(k, s) \exp (ikr) \, dk
\]

\[
= \frac{W_P W_{PS}}{(2\pi)^3} \int_0^\infty G_p(k, s) G_s(k, s) \exp (ikr) \, dk, \tag{17}
\]

where \( G_p(k, s) = G_p(-k, s) \) is used. Here, closing the contour of integration at infinities in the complex \( k \) plane, we adopt the technique of residue and branch cut integration as has been done for a single branch cut (Ishimaru 1978, p. 233). The branch points of \( G_p \) and \( G_s \) in the complex \( k \) plane are given by

\[
\nu(s)i \quad \text{and} \quad \nu(v)i.
\]

(18)

where

\[
\nu(s) = \frac{s + \eta}{\alpha}, \tag{19}
\]

We take branch cuts from these branch points to infinity on the imaginary \( k \) axis. The integral contour is taken as two quarter circles and along two branch cuts on the imaginary axis as schematically illustrated in Fig. 2. We know that the integral around the two quarter circles vanishes and there remains the integral between the branch points of \( G_p \) and \( G_s \) and that between the branch point of \( G_s \) and the imaginary infinity:

\[
E_{PS}(r, s) = \frac{W_P W_{PS}}{4 \pi \alpha r} \int \int \left[ \frac{1}{u} \tanh^{-1} \frac{u}{\gamma} \right] \exp [-\nu(\nu)i] \, du
\]

\[
+ \int_0^1 \left[ \frac{1}{u} \tanh^{-1} \frac{1}{u} + \tanh^{-1} \frac{1}{\gamma} \right] \exp [-\nu(\nu)i] \, du
\]

\[
= \frac{W_P W_{PS}}{4 \pi \alpha r^2} \int_0^\infty K_C \left( \frac{at}{r} \right) H \left( \frac{at}{r} - 1 \right)
\]

\[
\times \exp (-\eta x) \exp (-st) \, dx, \tag{20}
\]

where \( H \) is the step function and

\[
K_C(x) = \frac{1}{x} \tanh^{-1} \frac{x}{\gamma} \quad \text{for} \quad 1 < x < \gamma
\]

\[
= \frac{1}{x} \left( \tanh^{-1} \frac{1}{x} + \tanh^{-1} \frac{1}{\gamma} \right) \quad \text{for} \quad x > \gamma. \tag{21}
\]

Eq. (20) is written in the form of Laplace transformation, where the integral kernel gives the solution in time:

\[
E_{PS}(r, t) = \frac{W_P W_{PS}}{4 \pi \alpha r} K_C \left( \frac{at}{r} \right) H \left( \frac{at}{r} - 1 \right) \exp (-\eta t). \tag{22}
\]

In a similar way, for single \( SP \) scattering, we get

\[
E_{SP}(r, t) = \frac{W_P W_{SP}}{4 \pi \alpha r} K_C \left( \frac{at}{r} \right) H \left( \frac{at}{r} - 1 \right) \exp (-\eta t). \tag{23}
\]

In the case of \( PP \) and \( SS \) single scattering, according to the integral around one branch cut (Zeng et al. 1991), we obtain

\[
E_{PP}(r, t) = \frac{W_P W_{PP}}{4 \pi \alpha r} K \left( \frac{at}{r} \right) H \left( \frac{at}{r} - 1 \right) \exp (-\eta t)
\]

\[
E_{SS}(r, t) = \frac{W_P W_{SS}}{4 \pi \alpha r} K \left( \frac{at}{r} \right) H \left( \frac{at}{r} - 1 \right) \exp (-\eta t) \tag{24}
\]
where

\[ K(x) = \frac{2}{x} \tanh^{-1} \frac{1}{x} \quad \text{for} \quad x > 1. \quad (25) \]

The functions \( K(x) \) and \( K_e(x) \) were previously obtained as eq. (3) in Sato (1977a) and eq. (17) in Sato (1977b), respectively, based on the integral in the prolate spheroidal coordinate system. These functions \( K(x) \) and \( K_e(x) \) logarithmically diverge at \( x = 1 \) and \( x = \gamma \), respectively, as shown in Fig. 3, but, time integrals containing these singularities are convergent. \( K(x) \) shows that single PP scattering and SS scattering just transfer energy backwards; however, single SP scattering has a peak at the S-wave arrival when the receiver is located on the corresponding scattering shell, and single PS scattering has a peak when the hypocentre is located on the corresponding scattering shell (see Sato 1977b).

### 4 NUMERICAL SIMULATION

We can numerically evaluate the multiple scattering term \( E^m(r, t) \) by using the inverse Fourier transform instead of the inverse Laplace transform along the imaginary \( k \) axis:

\[
E^m(r, t) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} ds \exp(st) \times \left[ \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} dk k \exp(ikr)E^m(k, s) \right] \\
= \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} d\omega \exp(-i\omega) \left\{ \int_{-\infty}^{\infty} dk \exp(-ikr) \\
\times \frac{ik}{2\pi} E^m(k \rightarrow -k, z \rightarrow -i\omega) \right\}. \quad (26)
\]

As examples, we numerically synthesize time traces of energy density, where we suppose the ratio of \( P \)- and \( S \)-wave velocities \( \gamma = \sqrt{3} \) and no intrinsic absorption \( \zeta' = 0 \). So far, our model adopts a spherical radiation only, but we suppose the ratio of \( P \)- and \( S \)-wave radiations to be that of a point shear dislocation, \( W_S/W_P = 1.5 \gamma^2 \). That is, we characterize the scattering system by one parameter \( \eta \). 2-D FFT is done for 256 × 256 points for \((0-8), (0-8)\) in the normalized space–time \((\rho, \tau)\):

\[
\rho = \frac{\eta}{\alpha} \tau, \quad \tau = \eta t, \quad \epsilon(\rho, \tau) = \left( \frac{\alpha}{\eta} \right)^3 \left[ \frac{E(r, t)}{W_P + W_S} \right], \quad (27)
\]

where time and length are measured by the reciprocal of the mean free time and that of the mean free path, respectively. The single scattering term logarithmically diverges at \( \tau = \rho \) and \( \tau = \gamma \rho \) because of singularities of \( K(x) \) and \( K_e(x) \); however, the integral over a window containing a singularity is finite.

The first example is chosen for mathematical simplicity: with the four scattering parameters equal to each other, \( \xi_{PP} = \xi_{PS} = \xi_{SS} = \xi_{SP} = \eta/2 \); then, the cross-term \( G_{PS}G_S \) in the denominator of eq. (15) vanishes and the numerical calculation becomes simple. In the second example, we take a more realistic choice of parameters. In a random medium, the scattering attenuation calculated by the integral of a non-isotropic scattering cross-section provided by the Born approximation over scattering angle yields the scattering parameters which are used. In the case of an exponential autocorrelation function for the fractional fluctuation of velocity, the calculated ratios of scattering attenuations are

\[
\frac{Q^{-1}_{PS}}{Q^{-1}_{PS}} \approx 2 \quad \text{and} \quad \frac{Q^{-1}_{SP}}{Q^{-1}_{SP}} \approx 9
\]

in high frequencies (Sato 1984, Fig. 10). In order to calculate the single scattering term analytically, we can not violate condition (16). We, therefore, adopt the ratios: \( \zeta_{PP} = 2\eta/3, \quad \zeta_{PS} = \eta/3, \quad \zeta_{SS} = 9\eta/10 \) and \( \zeta_{SP} = \eta/10 \), which satisfy condition (16).

We plot the temporal changes in the normalized density \( \epsilon \) in Figs 4(a) and (b) at three different frequencies. Each arrival time of a P wave corresponds to the hypocentral distance. The energy density of multiple scattering here calculated is plotted by a broken curve, and that of direct and single scattering is plotted by a dotted curve. The source duration time is two samples, 1/16 in normalized time; the total energy density is plotted as a solid curve. In Fig. 4(b), the P-coda level is small and gradually increases with increasing lapse time. The direct wave energy rapidly decreases with increasing distance. This simulation gives a good qualitative explanation of the stable amplitude of the P-coda, as is often observed. In Fig. 4(a), we can clarify that the contribution of large SP scattering produces a more rapid increase of P-coda amplitude with increasing lapse time.

We can synthesize the energy density for any source time function by convolution. If we increase the source duration, the power of the direct P and S phases decrease, however, the envelope due to multiple scattering changes little. We show temporal traces for different source duration times in Fig. 5. Solid and dotted curves correspond to 1/32 (one sample) and 1/8 (four samples) in normalized time, respectively. The direct and single scattering contribution becomes much smaller as the source duration time increases, but the multiple scattering contributions are almost independent of source duration time. This simulation suggests that spindle-type seismogram envelopes might be explained by multiple scattering with conversions.
Figure 4. Temporal traces for the energy density (solid curves) at three different distances, where the source duration is two samples, 1/16 in normalized time: an analytical solution for the single scattering and the direct waves (dotted curves); a numerical solution by using the 2-D Fourier transform for multiple scattering of the order higher than, or equal to, two (broken curves). (a) $\xi_{pp} = \xi_{ps} = \xi_{ss} = \eta/8$; (b) $\xi_{pp} = 2\eta/3$, $\xi_{ps} = \eta/3$, $\xi_{ss} = 9\eta/10$ at $\xi_{sp} = \eta/10$.

5 CONCLUSIONS

We have mathematically formulated the scattering process with two propagation velocities and four isotropic conversion scattering modes by a simple set of integral equations and Green's functions on the basis of energy transport theory. The temporal change in the total energy is solved analytically, and the partition of the energy into $P$ and $S$ waves is found to be controlled by the conversion scattering parameters. The conservation of total energy in the case of no intrinsic absorption assures the mathematical consistency of the formulation. The analytical solutions for the single scattering energy density are obtained from a contour integral in the complex $k$ plane only when the total attenuation is the same for $P$ and $S$ waves. The analytical solution well clarifies the role of each of the scattering parameters. A hybrid simulation method for the energy density is proposed by adding analytical solutions for single scattering and a numerical simulation based on the 2-D FFT for multiple scattering. The stationary or the gradual increase of $P$-coda amplitudes commonly observed and spindle-type seismogram envelopes are qualitatively well explained by multiple scattering with conversions. The envelope shape characterized by multiple scattering is found
to be not very sensitive to the change in source duration time.

This formulation is mathematically consistent for any choice of scattering parameters. An advantage of our hybrid simulation method is the use of the analytical solution for the single scattering term, but the simulation fails when eq. (16) is violated. For such a case, we have to use the inverse Fourier transformation, not only for the multiple scattering energy density but for all the energy density. The convergence of the integral is slow and a more sophisticated technique is necessary for numerical integration (see Zeng 1993). The energy transport theory is found to be effective for the study of multiple conversion scattering processes; however, it does not mean that the isotropic scattering assumption is valid. We will have to include the non-spherical radiation as a point shear dislocation in the formulation. Recently, improving the conventional energy transport theory, Sato (1993b) proposed a formulation of a non-isotropic scattering process without conversion scattering in the 2-D case, where the non-isotropic scattering is expanded by the Fourier series with respect to scattering angle. The isotropic scattering is interpreted as the lowest term of the Fourier expansion. An extension to the 3-D case and scattering with $P-S$ conversions are required. When these tasks are completed, it will become possible to compare quantitatively seismogram envelopes observed and numerically simulated. In parallel with the development of the energy transfer approach for non-isotropic scattering, it will be necessary to model the variations of inhomogeneous media in a full elastic wave equation approach in order to overcome the point-like scattering assumption.

ACKNOWLEDGMENTS

The author is grateful to Brian Kennett for his careful help in improving the readability of the text. An anonymous reviewer’s comments were helpful in revising the manuscript with respect to the more realistic choice of scattering parameters for the numerical synthesis.

REFERENCES


Wu, R. S., 1985. Multiple scattering and energy transfer of seismic waves—separation of scattering effect from intrinsic...