Seismogram envelope inversion for the spatial distribution of high-frequency energy radiation from the earthquake fault: Application to the 1994 far east off Sanriku earthquake, Japan

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Abstract. Incoherency of high-frequency seismic waves leads to the idea to analyze seismogram envelopes disregarding phase information for the source study. We develop an inversion method for estimating high-frequency (above 1 Hz) energy radiation from the earthquake fault and site amplification factors from observed mean square S-wave seismogram envelopes. The inversion is executed by using the S-wave envelope Green function from the onset to coda for a point shear dislocation source in a scattering medium, which is formulated based on an extended version of the radiative transfer theory. The use of the envelope Green function enables us to estimate the spatial distribution of high-frequency energy radiation on a fault plane. This has been difficult by conventional waveform inversion methods. The rupture velocity and the duration time of energy radiation on each subfault are also estimated by grid search method. We apply this new method to the 1994 far east off Sanriku, northeastern Japan, earthquake ($M_w$ 7.7), which ruptured a 90-km segment of the plate boundary along the Japan Trench. Inverting observed seismogram envelopes at 10 strong-motion stations in Japan, it is concluded that above 90% of the high-frequency energy was radiated from the western half of the fault with the largest energy radiation near the deep-side edge. The rupture velocity is estimated to be 2.7 km/s. The estimated site amplification factors, ranging between 0.3 and 15, are consistent with those independently estimated by the coda normalization method.

1. Introduction

Study on generation and propagation of high-frequency seismic waves (above 1 Hz) is indispensable not only to clarify the earthquake source process but also to quantitatively predict strong ground motions. For relatively lower frequencies, conventional waveform inversions [e.g., Harzell and Heaton, 1983; Fukushima and Irikura, 1986] have been successfully applied for estimating the spatial distribution of fault slip. However, it is difficult to simply apply these schemes to high-frequency waves due to our poor knowledge of small-scale heterogeneities in the Earth medium and the difficulty in waveform fitting of observed seismograms that are abundant in incoherent wave trains.

To avoid these difficulties, a few studies attempted to use seismogram envelopes for revealing earthquake source process at high frequencies. Gusev and Pavlov [1991] deconvolved mean square (MS) envelopes of far-field $P$-wave velocity seismograms of the 1978 off Miyagi, Japan, earthquake ($M_7.6$), and estimated the location of a "short-period radiator", which corresponds to a centroid for high-frequency wave radiation. Cocco and Boatwright [1993] deconvolved an MS envelope of acceleration record and estimated the power rate function for one aftershock ($M_5.9$) of the 1976 Friuli earthquake. Kakehi and Irikura [1996] estimated high-frequency wave radiation areas on the fault of the 1993 Kushiro-Oki earthquake ($M_w$ 7.6) by using the root-mean-square envelope of acceleration seismograms. These three studies used seismograms of small earthquakes as empirical Green functions. Zeng et al. [1993] inverted MS envelopes of displacement seismograms from the 1989 Loma Prieta earthquake by using ray theoretically calculated Green functions and mapped the high-frequency source radiation intensity on the earthquake fault. These previous studies indicate that the location of the high-frequency source did not coincide with that of the low-frequency source.

However, there are some weak points in these schemes. Though the empirical Green functions naturally include propagation and site effects, records of a small event which has a similar hypocenter and focal mechanism to a target event are not always available.

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Source parameter of a target event can be estimated only with relative to those of a small event by using this method. The ray theoretical approach cannot deal with coda waves appearing in high-frequency records. It is hard to discriminate source term from propagation term, since observed coda energy may be regarded as source energy by this approach. Therefore it is necessary to construct a new method that can discriminate between the source, propagation, and site effects, and can resolve absolute source parameters for high-frequency seismic waves.

In this study we develop an inversion method to find the spatial distribution of high-frequency energy radiation and site amplification factors by minimizing the residuals between observed MS velocity seismogram envelopes and synthetic ones for each of octave-width frequency bands. Separation of source and site effects is conducted by fixing the site amplification factor at one hard rock site. Propagation effect is incorporated theoretically based on the radiative transfer theory in scattering media [e.g., Chandrasekhar, 1960; Wu, 1985]. Using scattering characteristics of the media in each octave-width frequency band, we synthesize the envelope Green function for the whole S-wave seismogram including coda energy by following Sato et al. [1997], which succeeded in introducing the four-lobe radiation pattern from a point shear dislocation source as an extension of the conventional radiative transfer theory [e.g., Zeng et al., 1991; Sato, 1995].

Then we apply a new inversion method to the 1994 far east off Sanriku, northeastern Japan, earthquake (MW 7.7; hereafter referred to as the off Sanriku earthquake) as a case study. This earthquake occurred on December 28, 1994, and ruptured a 90-km-width segment of the plate boundary between the subducting Pacific plate and the landward one. Analyzing high-frequency seismogram envelopes, which were recorded by many strong-motion seismometers distributed in the northeastern Japan (Figures 1 and 2), we estimate the spatial distribution of high-frequency energy radiation from this earthquake fault.

2. Method

2.1. Green Function for Envelope

First, we briefly summarize the energy propagation process in three-dimensional (3-D) scattering media after Sato et al. [1997]. We assume that point-like isotropic scatterers are distributed randomly and homogeneously in an infinite medium, where the background above 1.0 Hz.

<table>
<thead>
<tr>
<th>SAP</th>
<th>KUS</th>
<th>URA</th>
<th>HAK</th>
<th>AOM</th>
<th>HAC</th>
<th>MRK</th>
<th>OFU</th>
<th>TYM</th>
<th>ISN</th>
</tr>
</thead>
</table>

Above 1.0 Hz

Figure 1a. Velocity seismogram (E-W component) of the off Sanriku earthquake for high-pass filtered traces above 1 Hz. All the traces start from 40 s after the mainshock origin time and are normalized by their maximum amplitudes.
$S$-wave velocity $V$ is constant. We put a point shear dislocation source at the origin, where seismic wave energy of unit amplitude is impulsively radiated. In the framework of the radiative transfer theory, the energy density at a given place and time $E_G(x, t)$ is expressed by the following convolution integral:

$$ E_G(x, t) = R(x, t)G(x, t) + g_0 V \int_{-\infty}^{\infty} G(x - x', t - t')E_G(x', t')dx'dt', $$

where $x = (r, \theta, \phi)$ is the spherical coordinate and $g_0$ is the total scattering coefficient characterizing the scattering power per unit volume. The function $E_G$ works as the envelope Green function in the scattering medium. $R(x, t)$ is the radiation pattern of $S$-wave energy normalized as $\int R(x, t)dx = 4\pi$. The first term on the right-hand side is the coherent part corresponding to the direct wave. The second term is the whole scattered energy, which is given by integrating the contributions from the last scattering point $x'$ and at lapse time $t'$. The propagator function $G(x, t)$ is expressed as

$$ G(x, t) = \frac{\exp\left[-(g_0V + \eta)t\right]}{4\pi V r^2} \delta\left(t - \frac{r}{V}\right)H(t) $$

where $r = \|x\|$. This is characterized by geometrical spreading, time lag due to propagation, and exponential decay due to intrinsic absorption and scattering attenuation of seismic wave energy. Intrinsic absorption is written as $Q^{-1} = \eta/\omega$ for an angular frequency $\omega$.

For a point shear dislocation source with fault normal vector in the first axis and slip vector in the second axis, we can explicitly express $R(x, t)$ in terms of the spherical harmonics expansion up to the fourth order as

$$ R(\theta, \phi) = \sum_{n=0,2,4} \sum_{m=-n}^n a_{nm} Y_{nm}(\theta, \phi) $$

$$ = \sqrt{4\pi} Y_{0,0}(\theta, \phi) + \frac{5}{\pi} \frac{4\pi}{5} Y_{2,0}(\theta, \phi) $$

$$ - \frac{2}{\pi} \frac{4\pi}{9} Y_{4,0}(\theta, \phi) $$

$$ + \frac{\sqrt{280\pi}}{21} [Y_{4,\pm 4}(\theta, \phi) + Y_{4,\mp 4}(\theta, \phi)]. $$

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**Figure 1b.** Velocity seismogram (E-W component) of the off Sanriku earthquake for band-pass-filtered traces between 0.03 and 1 Hz. All the traces start from 40 s after the mainshock origin time and are normalized by their maximum amplitudes.
where $\theta$ is the zenith angle measured from the null axis (the third axis) and $\phi$ is the azimuth angle measured from the fault normal (the first axis).

We solve (1)-(3) for energy density $E_G(x, t)$ by using the Fourier transform in space, the Laplace transform in time, and the spherical harmonics expansion with respect to radiation angle. The energy density is written as a spherical harmonics expansion by using the expansion coefficients of radiation pattern:

$$E_G(x, t) = \sum_{n=-\infty}^{\infty} E_{G,n}(r, t) \sum_{m=-n}^{n} a_{nm} Y_{nm}(\theta, \phi),$$  \hspace{1cm} (4)

where

$$E_{G,n}(r, t) = \frac{e^{-(g_0 V + \eta)t}}{4\pi r^2 V} \delta \left( t - \frac{r}{V} \right) H(t)$$

$$+ \frac{g_0 e^{-(g_0 V + \eta)t}}{4\pi r V t} Q_n \left( \frac{V V}{V^2} + \frac{1}{2} \left( \frac{V V}{V^2} \right)^2 \right) H \left( t - \frac{r}{V} \right)$$

$$+ \frac{g_0^2 V^2}{2\pi r t} \int_{-\infty}^{\infty} d\omega \frac{e^{i\omega t}}{2\pi}$$

$$\cdot \int_{-\infty}^{\infty} dk \frac{e^{ikr}}{2\pi} k u_n(kr) \overline{G_n}(k, \omega) \overline{G_0^2}(k, i\omega) \frac{1 - g_0 V \overline{G_0}(k, i\omega)}{1 - g_0 V \overline{G_0}(k, i\omega)}. \hspace{1cm} (5)$$

The first term is the coherent term, the second is the single scattering term, and the third is multiple scattering term of the order $\geq 2$. $Q_n$ is the Legendre function of the second kind. Function $u_n(x)$ originates from the spherical Bessel function and is defined as

$$u_n(x) = \left( \frac{x}{2} \right)^n \frac{\Gamma(n + 1)}{\Gamma(n + \frac{3}{2})}, \hspace{1cm} (6)$$

Function $G_n(k, s)$ corresponds to the Laplace transform of the spherical Bessel function:

$$G_n(k, s) = \frac{1}{kV} \left( \frac{kV}{2(s + gV + \eta)} \right)^{n+1} \sqrt{\pi} \Gamma(n + 1) \Gamma(n + \frac{3}{2})$$

$$\cdot 2 F_1 \left[ \frac{n + 1}{2}, \frac{n + 2}{2}, n + 3 \right] - \left( \frac{kV}{s + gV + \eta} \right)^2, \hspace{1cm} (7)$$

where $\Gamma$ is Gamma function and $2F_1$ is Gauss's hypergeometric function.

We can numerically calculate $E_{G,n}(r, t)$ using the fast Fourier transform (FFT) algorithm over frequency and wavenumber for given three parameters, $g_0$, $V$, and $Q_s^{-1}$. The prominent character of the envelope Green...
function $E_G(r, t)$ is having a long tail which follows the direct wave and decays slowly due to scattering. The energy density faithfully reflects source radiation pattern immediately after the direct wave arrival. However, the energy density of a higher-order mode diminishes faster than that of a lower-order mode with increasing lapse time. Therefore only the lowest 0th mode corresponding to spherical source radiation dominates at a large lapse time.

2.2. Inversion Method

We present an inversion method to estimate the spatial distribution of energy radiation from the earthquake fault and site amplification factors by using the envelope Green function presented in section 2.1. We assume that a rupture starting from the hypocenter propagates with a constant rupture velocity of $V_r$ on the main fault, which is divided into several subfaults. When a rupture front passes through the $k$th subfault, the source energy $W_k$ is radiated from a point shear dislocation source on this subfault with a time history of $f_k(t)$. The integral of $f_k(t)$ over the transit time of the rupture front is normalized as 1. The radiated energy is multiply scattered in the course of propagation through a 3-D scattering medium, and reaches the $i$th station at the $j$th time (Figure 3). We further assume that the waves radiated from different subfaults are incoherent each other so that the energy density for the $i$th station and the $j$th time is composed by superposing the radiated energy from different subfaults. Then, we can express the theoretical energy density $C_{ij}$ as

$$C_{ij} = S_i^2 F_{ijk} W_k,$$

where

$$F_{ijk} = \int f_k(t') E_G(x_i - x_k, t_j - t') dt'.$$

$F_{ijk}$ is the convolution of the envelope Green function and the energy radiation time history. $S_i$ is the site amplification factor for velocity amplitude at the $i$th station. We estimate the values of $W_k$ and $S_i$ which minimize the residual between observed and synthetic envelopes in the following least squares sense:

$$\sum_i \sum_j \left( \frac{1}{\max_j O_{ij}} \right)^2 |O_{ij} - C_{ij}|^2 \rightarrow \min,$$

where $O_{ij}$ is the observed energy density. We normalize both the observed envelopes and the synthetic ones by the observed maximum value at each station to set the weight of all the stations equal. To simplify the inversion, we further assume that $f_k(t)$ is a box-car function with the same duration time of $\Delta t$ for all the subfaults.

Since equation (10) is nonlinear for the radiated energy $W_k$ and the site amplification factor $S_i$, we solve this equation iteratively by (1) assuming values of $V_r$ and $\Delta t$, (2) setting initial value of $S_i$ at 2 for all the stations, where 2 is a standard site amplification factor for velocity amplitude on the free surface, (3) solving equation (10) for energy radiation intensity $W_k$ by using the linear least squares method, (4) estimating the site amplification factors by fixing the energy radiation intensity calculated in step 3, and (5) iterating steps 3 and 4 until the residual between the observed and synthetic envelopes does not change with increasing iteration steps. We thus estimate the best fit values of $W_k$ and $S_i$ for various sets of $V_r$ and $\Delta t$. The final result is obtained by choosing the solution having the minimum residual among them.

3. Analysis of the Off Sanriku Earthquake

The off Sanriku earthquake occurred on December 28, 1994, at about 200 km east off the Pacific coast of northeastern Japan. At the nearest station Hachinohe (HAC in Figure 2), the maximum acceleration of about 600 Gal was recorded, three people were killed and about 800 people were wounded. Seismic intensity by the Japan Meteorological Agency (JMA) was 6 at HAC and 5 at AOM and MRK. The Harvard centroid moment tensor (CMT) solution shows the focal mechanism of a reverse fault type with a strike of 184°, a dip of 15°, and a rake of 70°, and the centroid located at 143.12°E, 40.41°N, 34 km in depth. The mainshock
hypocenter is located by the Observation Center for Prediction of Earthquakes and Volcanic Eruptions, Tohoku University (OCPEV) at 143.82°E, 40.46°N, and 13 km in depth, which is close to the Japan Trench (see Figure 2). Most of the aftershocks are confined to the region of about 170 km x 90 km, and their depths increase from 13 km to 50 km from the east to the west (Figure 4). The rupture proceeded westward from the initial rupture point near the Japan Trench [e.g., Sato et al., 1996; Kato and Takemura, 1996].

3.1. Data

We use seismograms recorded at 10 stations with hypocentral distance ranging from about 100 km to 400 km (Figure 2): nine stations of the JMA and one station of Tohoku University. The JMA stations are equipped with the three-component strong motion seismometer (JMA 87 type), which has a flat response to acceleration for frequencies between DC and 400 Hz. However, since the sampling frequency is 50 Hz, output signal is low-passed by the sixth-order Bessel filter (-3 dB at 10 Hz) for antialiasing (M. Hoshiba, personal communication, 1997). At Tsuyma (TYM in Figure 2), a three-component broadband seismometer (STS-2), which has a flat response to velocity between about 0.08 Hz and 30 Hz, is deployed. The sampling frequency is 80 Hz. Since this station is located on a hard rock of Mesozoic or older sandstone, we adopt TYM as a reference station for the estimation of site amplification factors.

We convert these records to velocity seismograms to obtain band-pass-filtered velocity seismograms for the four octave-width frequency bands of 1-2, 2-4, 4-8, and 8-16 Hz. Then squared velocity seismograms are calculated by adding three components for each station and each frequency band. We smooth them by taking running average over a time window of 5 s and resample the data at every 1.5 s from the S-wave onset to the lapse time of 200 s. The MS velocity seismogram thus constructed is multiplied by mass density (2.8 g/cm³) to get the energy density envelope which has a dimension of energy density (J/m³). Hereafter, this energy density envelope is simply referred to as envelope.

Figure 4 shows observed envelopes by solid lines. The duration of the observed ground motion varies a lot from station to station: as long as 50 s at TYM and as short as 10 s at HAC. The number of peaks ranges from one to three: one at HAC, AOM, HAK and three at KUS for 1-2 Hz.

3.2. Fault Model

We assume that the fault plane of the off Sanriku earthquake is in the region of 142°-144°E and 40.0°-40.8°N based on the aftershock distribution and the study of Nishimura et al. [1996]. The fault plane, dipping to the west, is set as shown in the E-W cross section of Figure 4 following the relocations of aftershock hypocenters by Umino et al. [1995]. The depth of the fault changes from 13 km at the eastern edge to 50 km at the western edge.

We divide the inclined fault plane into 32 subfaults, the size of which is roughly 20 km x 20 km. The strike and rake are assumed at 184° and 70°, respectively, for all the subfaults following the Harvard CMT solution. We change the dip angles from 0° at the eastern edge to 30° at the western edge based on the focal mechanism of aftershocks [Matsuzawa et al., 1995]. Each subfault breaks when the rupture front passes through it. We assume that the rupture proceeded from the initial rupture point at the east end with a constant rupture velocity $V_r$.

3.3. Calculation of Envelope Green Function

We need the values of S-wave velocity $V$, total scattering coefficient $g_0$, and intrinsic absorption $Q_i^{-1}$ for our study region to calculate synthetic envelopes. Constant S-wave velocity $V$ is estimated to be 3.9 km/s from the observed S-wave travel times of aftershocks. For the total scattering coefficient $g_0$ and the intrinsic absorption $Q_i^{-1}$, we adopt $g_0 = 0.005$ km$^{-1}$ (1-16 Hz), and $Q_i^{-1} = 0.0033$ to 0.001 (1-16 Hz) following the results of Hoshiba [1993] and Sakurai et al. [1995] for the
Figure 5a. Observed envelopes (solid line) calculated from the three-component MS seismogram envelopes corresponding to energy density and the best fit syntheses obtained by the inversion (dotted line) for 1–2 Hz frequency band.

Figure 5b. Same as Figure 5a for 2–4 Hz frequency band.

Figure 5c. Same as Figure 5a for 4–8 Hz frequency band.

Figure 5d. Same as Figure 5a for 8–16 Hz frequency band.
region in and near the present study area. The adopted values in this study are shown by solid circles in Figure 6. Substituting these parameters into equation (5), we calculate the synthetic envelopes for all the stations by using the FFT algorithm over frequency and wavenumber with grids of 256 x 256. The sampling interval of envelope is 1.5 s.

The assumption of constant V might cause errors in the travel time and emergent angle of the direct S wave. Hence we correct travel times by using station corrections listed in Table 1. We use emergent angle corrected by the layered velocity structure of Imanishi [1996] (Table 2) for each station-subfault pair.

As an example, we show the comparison of theoretical envelope Green functions and observed envelopes for an aftershock (December 30, 1994, 142.31°E, 40.22°N, 47.8 km in depth, $M_{JMA}$ 6.0) within 8-16 Hz band (Figure 7). The Harvard CMT focal mechanism of this after-shock is as follows: strike of 181°, dip of 25°, and rake of 64°. Theoretical envelope Green functions are calculated for parameters assumed in Figure 6 and are set to coincide with observed ones at 100 s for correcting source energy and site amplification factor. Theoretical envelope Green functions match observed envelopes well for not only this frequency band but also other frequency bands. It means that the contribution of surface waves to observed high-frequency envelopes is small, and it also validates the neglect of S wave to surface wave conversion in our theoretical modeling.

3.4. Results of Inversion

We solve equation (10) for the rupture velocity $V_r$ ranging from 2.1 to 3.6 km/s, and the duration time of energy radiation $\Delta t$ from 3 to 15 s using an algorithm of nonnegative least squares problem developed by Lawson and Hanson [1974]. We put a constraint that four adjacent subfaults radiate the same amount of high-frequency energy, so that unknown parameters of energy radiation are estimated for eight blocks, each of which is composed of four adjacent subfaults. Those blocks are labeled by numbers 1 to 8 as shown in Figure 2. As another constraint to estimate the absolute values of radiated energy, we fix the site amplification factor for the hard-rock site TYM at 2 during the iteration. The best fit solution is obtained by about 70 iterations for each of the parameter sets.

Figure 8 shows residuals between the observed envelopes and the synthetic ones for all the $V_r - \Delta t$ sets. Each numeral appended to contour line is the residual defined in equation (10). For all the frequency bands, the rupture velocity is consistently estimated to be 2.7 km/s. The best fit duration time of energy radiation is 6 s for 1-2 and 2-4 Hz and is 9 s for 4-8 and 8-16 Hz. However, the resolution of $\Delta t$ is not high enough to definitely distinguish 6 s from 9 s, so that we adopt

<table>
<thead>
<tr>
<th>Table 1. Station Corrections for Travel Time</th>
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<tbody>
<tr>
<td>Station</td>
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<tr>
<td>AOM</td>
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</table>

<table>
<thead>
<tr>
<th>Table 2. Horizontally Layered Structure for Calculating Travel Times and Correcting Emergent Angles</th>
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</thead>
<tbody>
<tr>
<td>Layer</td>
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<tr>
<td>-------</td>
</tr>
<tr>
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<tr>
<td>2</td>
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<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
</tbody>
</table>

After Imanishi [1996].
found that the maximum amplitude of the synthetic envelope fits to that of observed one within 20% for most stations. However, there are some inconsistencies between them. For example, the observed envelope at KUS for 1–2 Hz has three peaks, whereas the synthetic one cannot well explain such behavior.

The best fit site amplification factors $S_i$ are shown by open squares in Figure 9, where the values are normalized by the value at the hard-rock site TYM. The site amplification factors range between 0.3 and 15, most of which are larger than 1, and show strong frequency dependence. Small dots designate results obtained by the coda normalization method, the logarithmic average of which is shown by a solid circle. These results are discussed in the section 4.2.

Figure 10 shows the best fit result of the radiated high-frequency energy. Total amount of radiated energy is $3.4 \times 10^{15}$ J for 1–2 Hz, $5.1 \times 10^{15}$ J for 2–4 Hz, $5.3 \times 10^{15}$ J for 4–8 Hz, and $1.4 \times 10^{15}$ J for 8–16 Hz, respectively. For all of the four frequency bands (1–2, 2–4, 4–8, 8–16 Hz), more than 90% of the total energy is radiated from the western half of the fault. The largest energy is radiated from the westernmost block (block 7), the amount of which reaches about 50% of the total energy.

4. Discussion

4.1. Stability of Inversion Results

We estimate the significance limit for the best fit solution of radiated energy $W_k$ and site amplification factor $S_i$.

Figure 8. Contour maps of normalized residuals (the left-hand side of equation (10)) for rupture velocity $V_r$ and the duration time $\Delta t$ in four frequency bands. Solid circle indicates the best fit point where the minimum residual is obtained.

Figure 9. Frequency dependence of site amplification factors ($S_i$) relative to the values at hard-rock site TYM. Open squares denote our inversion results. Small dots are results obtained by the coda normalization method, and solid circles are logarithmic average of them for each station.
The best fit solution indicates extremely large energy radiation from the western edge of the fault. In order to check the reliability of this result, we solve equation (10) by adding another two blocks (eight subfaults) to the west of blocks 7 and 8. This inversion results in small energy radiation from these appended subfaults with a value of about 5% of $W_7$ or $W_8$. Therefore the fault model we used is adequate, and the maximum energy radiation from the western edge is not an artificial result.

To evaluate the effect of the total scattering coefficient $g_0$ and the intrinsic absorption $Q^{-1}$ on the final result, we execute inversion by changing these two quantities within 10%. Though variation of $g_0$, which controls $S$-wave coda excitation, changes the radiated energy from each subfault within about 20%, we do not find any systematic change in the inversion result. The largest energy is still radiated from the northwestern edge, and the total amount of radiated energy and site factors do not have large changes (within 10% and 30%, respectively). For the variation of $Q^{-1}$, total radiated energy changes up to ±60%: larger radiated energy is obtained for larger $Q^{-1}$. However, the spatial pattern

<table>
<thead>
<tr>
<th>Block</th>
<th>1–2 Hz</th>
<th>2–4 Hz</th>
<th>4–8 Hz</th>
<th>8–16 Hz</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>0.003±0.027</td>
<td>0.016±0.021</td>
<td>0.011±0.017</td>
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</tr>
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<td>3</td>
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<td>4</td>
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<tr>
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<td>0.715±0.054</td>
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<tr>
<td>6</td>
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<td>1.069±0.070</td>
<td>0.725±0.041</td>
<td>0.126±0.007</td>
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<tr>
<td>7</td>
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<td>6.039±0.535</td>
<td>8.055±0.550</td>
<td>2.383±0.146</td>
</tr>
<tr>
<td>8</td>
<td>4.274±0.274</td>
<td>4.562±0.266</td>
<td>3.455±0.182</td>
<td>0.619±0.033</td>
</tr>
</tbody>
</table>

Energy radiation in units of $10^{14}$ J.
of $W_i$ on the fault and the site factors $S_i$ do not change significantly.

### 4.2. Site Amplification Factor

We estimate site amplification factors by using coda normalization method following Kato et al. [1995] and compare them with the inversion result to check the reliability of our method. Coda normalization method has been thought to be a stable way of estimating site amplification factors, since coda waves are for the most part composed of scattered S waves having various incident angles and the distribution of their energy is thought to be spatially uniform [e.g., Tsufuji, 1978; Phillips and Aki, 1986].

We use the observed MS velocity seismograms of the mainshock and two large aftershocks (December 30, 1994, for the JMA 7.0 aftershock, and 127.5 s for another aftershock). Taking the hard-rock site TYM as a reference station, we evaluate site amplification factors for the other nine stations for the four frequency bands. Results are shown by small dots in Figure 9. Solid circles are the logarithmic average for each station and frequency band. Results indicate extremely large amplification factors at KUS: the site amplification factor for the 8-16Hz frequency band is the largest among the all JMA stations (solid square). However, inversion results are lower than those determined by coda normalization method at a few stations (e.g., above 4Hz for HAC). The spatial heterogeneity of attenuation ($Q_i^{-1}$) structure may cause such a disagreement. In the present analysis, we assumed an identical site amplification factor so as to adjust the overestimated amplitude to the observed amplitude. Nonlinear site effects, which reduce the site response for strong motions, may also cause the disagreement of site amplification factors. For example, Chin and Aki [1991] showed that the site amplification factor at the time of the 1989 Loma Prieta earthquake was lower than that expected from the coda normalization method for stations on the sedimentary layer. Such a nonlinearity was clear for acceleration exceeding 200-300 Gal. As for the off Sanriku earthquake, the maximum acceleration was as large as 600 Gal at HAC. Even at AOM, more than 200 Gal was recorded. Therefore nonlinear site effect may have appeared at these stations during the strong ground shaking due to the off Sanriku earthquake.

### 4.3. High-Frequency Source Energy Radiation

Our inversion result shows that high-frequency energy is mainly radiated from the western half of the fault (Figure 10a). This is consistent with the result of travel time analysis that a high-frequency subevent was identified at the northern edge of the fault [Sato et al., 1996]. On the other hand, Nishimura et al. [1996] and Nakayama and Takeo [1997] reported that larger slip was concentrated near the center of the fault (Figure 10b). Aftershock activity was relatively low in the region of the large slip, and most of aftershocks occurred surrounding that region (see Figure 11).

Some previous studies [Zeng et al., 1993; Kakehi and Irikura, 1996] found a similar complementary relation between the locations of high-frequency source and low-frequency source. These results are consistent with the theoretical studies [e.g., Madariaga, 1976, 1977], which show high-frequency waves are radiated from the area where rupture stops or the slip velocity changes. However, a different feature was seen for the case of the 1968 off Tokachi earthquake ($M_W$ 8.2) (hereafter referred to as the off Tokachi earthquake), which occurred 26 years before the off Sanriku earthquake with partial overlap of the rupture regions (Figure 12). The slip distribution of the off Tokachi earthquake was investigated by waveform inversion of Rayleigh waves (10-25 s in period) [Mori and Shimazaki, 1985], and two large slip regions of more than 2 m were found at the northern and western edges of the fault (Figure 12). Two high-frequency subevents were also located at the northern and western edges of the source area (solid

### Table 4. Site Amplification Factors for Squared Velocity Amplitude

<table>
<thead>
<tr>
<th>Station</th>
<th>1-2 Hz</th>
<th>2-4 Hz</th>
<th>4-8 Hz</th>
<th>8-16 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>AOM</td>
<td>5.17±1.51</td>
<td>2.01±0.56</td>
<td>0.88±0.23</td>
<td>0.47±0.12</td>
</tr>
<tr>
<td>HAC</td>
<td>0.83±0.27</td>
<td>1.73±0.55</td>
<td>1.14±0.32</td>
<td>0.48±0.13</td>
</tr>
<tr>
<td>HAK</td>
<td>3.75±1.16</td>
<td>2.46±0.78</td>
<td>0.70±0.23</td>
<td>0.40±0.13</td>
</tr>
<tr>
<td>ISN</td>
<td>1.84±0.57</td>
<td>1.60±0.47</td>
<td>1.90±0.50</td>
<td>2.69±0.65</td>
</tr>
<tr>
<td>KUS</td>
<td>3.34±1.10</td>
<td>7.02±2.28</td>
<td>9.69±3.02</td>
<td>12.25±3.73</td>
</tr>
<tr>
<td>MRK</td>
<td>1.97±0.54</td>
<td>4.50±1.22</td>
<td>1.43±0.36</td>
<td>1.00±0.25</td>
</tr>
<tr>
<td>OFU</td>
<td>1.18±0.39</td>
<td>1.39±0.39</td>
<td>2.17±0.53</td>
<td>1.86±0.43</td>
</tr>
<tr>
<td>SAP</td>
<td>3.42±0.95</td>
<td>1.76±0.56</td>
<td>0.83±0.24</td>
<td>0.56±0.17</td>
</tr>
<tr>
<td>URA</td>
<td>4.05±1.24</td>
<td>4.17±1.21</td>
<td>1.89±0.54</td>
<td>0.96±0.26</td>
</tr>
</tbody>
</table>
circles in Figure 12) by travel time analysis of large-amplitude later phases found on near-field seismograms [Nagamune, 1969; Mori and Shimazaki, 1984].

So far it is not clear why such an opposite relation between the high-frequency and low-frequency sources occurred for these two earthquakes that took place at nearly the same place. It is interesting to note that block 7 in Figure 2 radiated large seismic wave energy in both of the successive large earthquakes. This particular portion of the plate interface may have an inherited physical property that generates large energy for high-frequency seismic waves.

5. Conclusion

We developed a new inversion method to estimate the spatial distribution of high-frequency energy radiation from an earthquake fault and site amplification factors by fitting S-wave MS velocity seismogram envelopes. Constant rupture velocity and constant duration time of energy radiation for each subfault are also determined by the grid search method. An extended version of the radiative transfer theory is used for synthesizing the envelope Green function for whole S-wave seismograms including coda part. The envelope Green function obtained is described by only three parameters of a scattering medium: S-wave velocity \( V_s \), total scattering coefficient \( g_0 \), and intrinsic absorption \( Q_s^{-1} \).

We applied this new inversion method to the 1994 off Sanriku earthquake (\( M_w 7.7 \)). Conducting inversion of observed envelopes at 10 stations for four frequency bands of 1–2, 2–4, 4–8, and 8–16 Hz, we obtained the following results. The total amount of radiated energy is \( 3.4 \times 10^{15} \) J for 1–2 Hz, \( 5.1 \times 10^{15} \) J for 2–4 Hz, \( 5.3 \times 10^{15} \) J for 4–8 Hz, and \( 1.4 \times 10^{15} \) J for 8–16 Hz.

Figure 11. Aftershock distribution of the off Sanriku earthquake (December 28, 1994, to January 10, 1995; after OCPEV). Shaded region is block 7, where the largest high-frequency energy was radiated (this study). Star and diamond are the initial rupture point and the Harvard CMT centroid of the off Sanriku earthquake, respectively. Triangle is the epicenter of the largest aftershock of the off Sanriku earthquake. Circle is a high-frequency subevent of the off Tokachi earthquake [Mori and Shimazaki, 1984].

Figure 12. Slip distribution of the off Tokachi earthquake [after Mori and Shimazaki, 1985]. The solid circles denote high-frequency subevents determined by Mori and Shimazaki [1984]. Shaded area represents the fault plane of the off Sanriku earthquake.

more than 90% of the total high-frequency energy was radiated from the western half of the fault. The maximum amount of energy was radiated from the westernmost edge, where the fault slip was rather small. Estimated site amplification factors range between 0.3 and 15. These values are comparable to those independently estimated by the coda normalization method.

For the high-frequency range above 1 Hz, conventional waveform inversion is difficult to apply, since we have little information for small-scale heterogeneities in the Earth medium. The new inversion method we developed will provide a new tool for revealing the spatial and temporal characteristics of high-frequency radiation from an earthquake fault.

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References


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