Scale Dependence of Apparent Stress for Earthquakes along the Subducting Pacific Plate in Northeastern Honshu, Japan

by Tsutomu Takahashi, Haruo Sato, Masakazu Ohtake, and Kazushige Obara

Abstract  Apparent stress, which quantifies the ratio of high-frequency spectral amplitude to the direct current level of source displacement spectra, is known to reflect the source rupture process. Whether this parameter depends on the seismic moment has not yet been well established because of the difficulties in the reliable estimation of high-frequency source spectra. To overcome this problem, we estimate $S$-wave source spectra by using site amplification factors and attenuation factor $Q_s^{-1}$, which are systematically measured by the coda normalization method. Analyzing horizontal components of velocity seismograms recorded at 70-borehole seismic stations of Hi-net (National Research Institute for Earth Science and Disaster Prevention) in northeastern Honshu, Japan, we estimate site factors and $Q_s^{-1}$ in the 0.5–32 Hz frequency band. By using these factors, we evaluate the source spectra of 225 small to moderate earthquakes that occurred in and around the subducting Pacific plate. Site-amplification factors we obtained strongly depend on frequency, and the frequency dependence clearly changes with the lithology and geologic age of rock. Obtained $Q_s^{-1}$ values decrease with frequency in proportion to the reciprocal of frequency. The apparent stresses estimated from $S$-wave spectra clearly increase from $10^4$ to $10^7$ Pa with the seismic moment increasing, which cannot be attributed to the limited-frequency band and other artificial causes. The power of scale dependence is estimated as 0.39–0.44 for the seismic moment range from $10^{11}$ to $10^{17}$ N m. Some of the scale dependence of apparent stress is attributed to the scale dependence of Brune’s stress drop.

Introduction

Apparent stress is defined as the ratio of seismic energy to seismic moment multiplied by the rigidity of medium (Aki, 1966). This parameter represents the relative excitation of the high-frequency component to the direct current (DC) level of the source displacement spectra and is thought to be affected by the frictional properties of the fault and the dynamic rupture process. For example, lower dynamic frictional stress will lead to higher apparent stress even for events of the same seismic moment if source ruptures start with the same initial stress (Kikuchi, 1992; Kanamori and Heaton, 2000; Mori et al., 2003), and the lubrication effect by gouge or water on the fault may cause high apparent stress for large earthquakes (Brodsky and Kanamori, 2001). The accurate knowledge of apparent stress may help us to understand the rupture process by providing essential information on the frictional properties of the fault.

In the past decades, many researchers measured the apparent stress in various regions in the world. Most studies of small seismic-moment earthquakes showed that the apparent stress clearly increases with the seismic moment increasing (Gibowicz et al., 1991; Abercrombie, 1995; Jost et al., 1998; Mayeda and Walter, 1996). The power of scale dependence is roughly 0.2–0.4 for the seismic moment range of $10^{10}$–$10^{15}$ N m. For larger earthquakes, on the other hand, the value of apparent stress becomes nearly independent of seismic moment (Kanamori et al., 1993; Mayeda and Walter, 1996; Izutani and Kanamori, 2001). However, some of the past studies ignored the site-factor correction or assumed the frequency independence of attenuation factor $Q_s^{-1}$. These assumptions do not agree with observed facts (Phillips and Aki [1986] for site factors, and Aki [1980] for $Q_s^{-1}$ values) and may cause a systematic error in source spectra estimation especially for the high-frequency range. Ide and Beroza (2001) argued that the underestimation of these effects, especially for small earthquakes, could be a possible origin of the scale dependence of apparent stress. Recently, careful evaluations of apparent stress of earthquakes in Japan were reported by Kinoshita and Ohike (2002) and Jin and Fukuwama (2005). They showed relatively weak scale dependence for small seismic-moment events. However, the simultaneous inversion method used for separating the site-amplification factor from observed spectra in Kinoshita
and Ohike (2002) may contain some trade-offs between source and site factors. The coincidence of $S$-wave attenuation $Q_s^{-1}$ with coda attenuation $Q_c^{-1}$ assumed in Jin and Fukuyama (2005) is not fully confirmed yet, even though $Q_c^{-1}$ showed good agreement with $Q_s^{-1}$ in some regions (Hoshiba, 1993). We must make more careful analysis with adequate values of $Q_s^{-1}$ and site factors without trade-offs between seismic source spectra and those parameters.

The $Q_s^{-1}$ value has been estimated in various regions by using various methods: simultaneous inversion solving with source and site factors (Kinosita, 1994; Petukhin and Irikura, 2000; Nakamura and Uetake, 2002), the coda normalization method (Aki, 1980; Yoshimoto et al., 1993), and multiple lapse time window analysis (Fehler et al., 1992; Hoshiba, 1993; Jin et al., 1994). Most of these authors reported clear frequency dependence of $Q_s^{-1}$, especially for frequencies higher than 1 Hz. However, their definitions of $Q_s^{-1}$ were slightly different from each other. For example, Yoshimoto et al. (1993) estimated $Q_s^{-1}$ value from the maximum amplitude decay with travel distance. On the other hand, Nakamura and Uetake (2002) evaluated $Q_s^{-1}$ values from the decay of power spectra measured in a rather long time window. Knowing that seismic wave scattering causes peak-amplitude decay and broadening of wavelet in inhomogeneous earth media (Sato, 1989; Saito et al., 2002), we need to use the $Q_s^{-1}$ evaluated from the energy decay with travel distance to take into account the contribution of diffracted and multiple forward-scattering waves in a finite-length time window.

Site-amplification factor is defined as the wave amplification effect by the local structure just beneath a seismic station. There are many studies on the evaluation of this factor with the following two methods: using coda waves (Aki, 1969; Aki and Chouet, 1975; Tsujiura, 1978; Phillips and Aki, 1986; Kato et al., 1995), and using direct $S$ wave (Takehida et al., 1991; Uetake and Ikeura, 2002). Phillips and Aki (1986), estimating site factors at about 150 stations in California, showed that the site factor depends on frequency and that the frequency dependence is closely related to the lithology of rock on which a seismometer is installed. Similar dependence was also found in the other works listed here. According to their results, site-amplification factors at sedimentary rock sites depend on the deposit age at lower frequencies ($\sim$1.5 Hz). On these sites, site factors in high frequency show strong attenuation. These variations of site factors play an important role in the estimation of seismic source spectra, especially for small earthquakes.

Based on this background, it is clear that we must use adequate attenuation factors and site-amplification factors to estimate source spectra and apparent stress more accurately. In this study, we evaluate $Q_s^{-1}$ and site-amplification factors independently by using the coda normalization method. This method is appropriate for our analysis because it is free from the source effect and does not require any assumption on the frequency dependence of attenuation and site factors. In the estimation of $Q_s^{-1}$ values, we take into account the geometrical spreading factor in 1D velocity structure. By using these factors, we estimate source spectra without a priori assumption of source spectral shape. Based on the appropriate values of apparent stress obtained from these source spectra, we discuss the scale and depth dependence of apparent stress.

Data

We use velocity waveform data of 225 small and moderate earthquakes observed at Hi-net (National Research Institute for Earth Science and Disaster Prevention [NIED]) stations in the fore-arc side of northeastern Honshu, Japan. In this region, the Pacific plate is subducting beneath the continental plate from east to west (Hasegawa et al., 1994, 2000). The events analyzed in this study occurred in and around this oceanic plate. In this data set, both interplate and intraplate earthquakes are included because types of small earthquakes are difficult to distinguish. The events in the lower plane of the double seismic zone (Hasegawa et al., 1978) are excluded because the weak attenuation in the subducting oceanic slab (Umino and Hasegawa, 1984) is not considered in our $Q_s^{-1}$ estimation. We use hypocenter data determined by the Japan Meteorological Agency (JMA). The earthquake magnitude range of the events is from 1.8 to 5.5 and the focal depth range is from 32 to 120 km. Hypocenters of earthquakes and 70 Hi-net stations used in this study are shown in Figure 1 by circles and small squares, respectively.

The Hi-net is composed of about 600 seismic stations (Obara, 2002). Each station we used is equipped with a three-component velocity-type seismometer with natural frequency 1 Hz at the bottom of borehole. The damping factor of the seismometer is 0.7. The depths of most boreholes we used are a few hundred meters or about 1 km. Subsurface geology varies from station to station. One station (N.SNDH, shown in Fig. 1) with a depth of 1206 m is used as a reference station in site-factor estimation. Geology of this reference station is characterized as follows: young sedimentary layers (Quaternary and Neogene) extend from the surface to 250 m in depth, and hard sedimentary rock in the Triassic extends from 250 m to the bottom of the borehole. We use horizontal components of velocity seismograms recorded at a 100-Hz sampling rate. The recording system has a flat response from 2 to 30 Hz. We deconvolved the digital data by the recording-system response, which is shown in the Hi-net web-site as a preprocessing.

We only use waveform data of source-station pairs both located in the fore-arc side of northeastern Japan. The boundary of fore- and back-arc side, which is almost the same as the volcanic front (Sugimura, 1960), is plotted by a broken line in Figure 1. In contrast to the fore-arc side, waveforms in the back-arc side are characterized by strong attenuation and scattering (Umino and Hasegawa, 1984; Obara and Sato, 1995). Therefore, the uniform distribution of coda waves as the basic assumption for the coda normalization
analysis is broken if both fore-arc and back-arc areas are mixed.

Method

Observed S-wave velocity spectrum of an earthquake at the $i$th station at frequency $f$ can be represented as

$$v_i(f) = R_{rhi}s(f) \frac{\exp(-\pi f Q_s^{-1}(f)t_{ai})}{r_i^{(\Delta_i,h)}} G_i(f) F_s, \quad (1)$$

where $R_{rhi}$ is the radiation pattern of S wave, $r_i$ is the hypocentral distance, $Q_s^{-1}$ is the attenuation factor, $t_{ai}$ is the S-wave travel time, $G_i(f)$ is the site-amplification factor, and $F_s$ is the reflection effect from the free surface. Exponent of geometrical spreading factor $\gamma(\Delta_i, h)$ is calculated from the S wave’s 1D velocity structure (see Fig. 2), which is used in the routine hypocentral determination by the Research Center for Prediction of Earthquakes and Volcanic Eruptions, Tohoku University (Hasegawa et al., 1978). Parameters $\Delta_i$ and $h$ are the epicentral distance and the focal depth, respectively. Velocity source spectrum is related to the Fourier transform of seismic-moment rate function $\dot{M}(f)$ as

$$S(f) = \frac{2\pi f}{4\pi \rho \beta^2} \dot{M}(f), \quad (2)$$

where $\rho$ and $\beta$ are density and S-wave velocity at the source, respectively.

We assume the radiation pattern of the S wave as the root mean square (rms) value of a point shear dislocation $R_{rhi}^{rms} = \sqrt{2}/5$. Velocity source spectrum $S(f)$ is estimated as the square root of the average of the power spectra over $N$ stations as

$$S(f) = \frac{1}{R_{rhi}^{rms}} \left\{ \frac{1}{N} \sum_{i=1}^{N} \left( \frac{v_i(f) r_i^{(\Delta_i,h)}}{G_i(f) F_s \exp(-\pi f Q_s^{-1}(f)t_{ai})} \right)^2 \right\}^{1/2}. \quad (3)$$

Although the source duration time is short for small earthquakes, observed S-wave duration becomes long by diffraction and scattering effect as the travel distance increases.
To exclude the strongly distorted data due to scattering, we limit the epicentral distance to less than 100 km. We fix the time window length as 6.0 sec from the time 0.5 sec before S-wave onset, which is long enough to include the maximum amplitude in the previously selected distance range. This time window is also long enough to contain the reflected wave from the ground surface. For example, the two-way time of reflected S waves at the deepest borehole (N.SNDH) is about 1.7 sec. To take this time window effect into account, we assume the reflection effect $F_r$ to be 2 for all frequency bands at all the stations.

From the source spectrum estimated by equation (3), we evaluate the apparent stress as

$$\sigma_{ap} = \mu E_s,$$

where rigidity $\mu$ is assumed as 67 GPa because its variation is small in our depth range (30–120 km) and because it is 66–68 GPa according to the preliminary reference earth model (PREM) (Dziewonski and Anderson, 1981). Seismic energy $E_s$ is estimated from the integrated of the squared source velocity spectrum

$$E_s \approx 8\pi \rho \beta \int_{f_{min}}^{30 \text{ Hz}} |S(f)|^2 df,$$

where the lower bound of the integral $f_{min}$ is varied from 0.5 to 2.0 Hz to exclude microseisms. We estimate seismic moment $M_0$ and corner frequency $f_c$ by fitting the omega square model (Brune, 1970),

$$S(f) = \frac{1}{4\pi f^2} \frac{M_0}{1 + (ff_c)^2},$$

to the obtained source displacement spectrum, where $\rho$ and $\beta$ are assumed as 3.3 g/cm$^3$ and 4.3 km/sec, respectively. These values are based on the PREM model and the velocity structure shown in Figure 2. Note that the evaluation of seismic moment $M_0$ is not sensitive to the shape of assumed source spectra model, and seismic energy $E_s$ is estimated without any specific source models.

Site-Amplification Factors

We briefly describe the coda normalization method for the site-amplification factor estimation (Tsujiura, 1978; Phillips and Aki, 1986). As the lapse time measured from the source origin time becomes longer than twice the direct S-wave travel time $t_t$, coda wave amplitude becomes free from the radiation pattern and the temporal decay of coda amplitude is independent of epicentral distance (Aki and Chouet, 1975). Coda wave spectral amplitude at frequency $f$ at the $i$th station at lapse time $t_s (t_s \geq 2 \cdot t_t)$ can be represented as

$$A_i(f, t_s) = S(f)C(f, t_s)G_i(f),$$

where $C(f, t_s)$ is the coda excitation factor characterizing medium heterogeneity. By taking the ratio of coda spectral amplitudes at different stations for the same event at the same lapse time $t_s$,

$$A_i(f, t_s) = \frac{S(f)C(f, t_s)G_i(f)}{S(f)C(f, t_s)G_j(f)} = \frac{G_i(f)}{G_j(f)},$$

we can obtain the relative site-amplification factor.

The uniform distribution of coda waves is the basic assumption of this method. From the coda amplitude distribution of various events, the relative coda amplitudes are found to be almost independent of source locations when epicentral distances are less than 150 km. Under this condition, the assumption in equation (8) is adequate for our case. By taking the logarithm of equation (8) for many pairs of stations, we estimate relative site-amplification factors to a fixed reference station with the singular value decomposition method. To secure the stability of estimation, we do not fix $t_t$ at any particular time but sample many $A_i$ values from various points of the coda wave following the method of Kato et al. (1995). A time window of 10 sec length is shifted to a later part by 5 sec from the initial time window, of which the beginning is set at 2$t_t$ at the farther station of the pair.

In the estimation of relative site-amplification factors, we use the following procedure. Dividing the whole station pairs into seven clusters (see Fig. 3a), we estimate site factors as relative values to a temporary reference station in each cluster. By setting some stations to be shared by two clusters to connect the results in each cluster, we finally calculate all the site-amplification factors as relative values to one reference station. The deepest borehole station N.SNDH (1206 m in depth, see Figs. 1 and 3a) is selected as the reference station at which the lithology of rock at the bottom is hard sandstone in Triassic. The site-amplification factor of the reference station is assumed as 1 for all frequency bands.

Site factors are estimated in 12 narrow-frequency bands from 0.5 to 32 Hz, each of which has a half-octave bandwidth. Results are shown in Figure 3b–g as the average values for six frequency bands. At low frequencies, site-amplification factors are high along the middle of the island arc. Along the Pacific coast in northern part of the study area, site factors have low values of the same order as the reference station. At high frequencies, strong attenuation is observed at stations showing high amplification in low-frequency ranges. This spatial variation of site factors shows clear relation to the geologic and lithological condition of rock. We classify the site factor into six groups based on...
lithology and geologic age of the rock as shown in Figure 4. The information of rock based on logging data was taken from the KiK-Net (NIED) web site, where the KiK-Net is a strong-motion observation network sharing boreholes with the Hi-net. The number of stations shown in Figure 4 is 64, which is less than the total number of stations because of the lack of logging data at some stations. We find that the site factors strongly depend on frequency and that the frequency dependence varies according to the lithology and geologic age of the rock. The frequency dependence is large for Quaternary sedimentary rock sites and small for igneous, metamorphic sites of Jurassic or before.

In the group of metamorphic/igneous rock sites of Neogene-Cretaceous period, most of the stations show weak frequency dependence. However, only one station shows strong frequency dependence. According to the logging data, this station is on dacite rock in Neogene with an $S$-wave velocity of about 0.6 km/sec. Because the $S$-wave velocity of other stations in this group is about 2.0–3.0 km/sec, this steep spectral decay might be due to the weathering effect of the rock.

The lithology dependence of estimated site factors is almost similar to the results of past works (Phillips and Aki, 1986). Uetake and Ikeura (2002) estimated the site factors in the Fukushima area (the middle part of our study area in the north–south direction) for the surface stations of the strong-motion observation network, the K-Net (NIED). Their results also show strong frequency dependence for most of the stations. The difference with our result is the strong amplification at high frequencies for the hard rock site in Cretaceous or Paleogene. They mentioned that it might be due to the weathering effect because the $S$-wave velocity of the
rock is not so large at these sites. The relatively stable site factor in our result may be due to the little weathering effect at borehole sites.

Attenuation Factor

Attenuation factor $Q_s^{-1}$ is estimated from the $S$-wave energy decay with travel distance. We modify the coda normalization method given by Yoshimoto et al. (1993) to take into account the geometrical spreading factor for the 1D velocity structure. By using equations (1) and (7), we obtain

$$\ln \left( \frac{r_s^2(t_s)}{A_s^2(f,t_s)} \right) = -2\pi f Q_s^{-1} t_{si} + \ln R_{0di}^2 - \ln C^2(f,t_s) + \ln F^2, \quad (9)$$

where $\tilde{v}_s^2$ and $\tilde{A}_s^2$ are the average power spectral amplitude in narrow frequency band around frequency $f$ at the $i$th station for the $S$ wave and coda wave, respectively. We fix $t_s$ as 80 sec, and the time window length of coda wave as 10.0 sec. $S$-wave travel time $t_s$ is less than a half of $t_s$, and the time window length of $S$ wave is fixed as 6.0 sec, which is the same as the window length used in the source spectra estimation. Assuming that the second to fourth terms of the right-hand side of equation (9) are independent of source parameters (Yoshimoto et al., 1993), we can simplify equation (9) as

$$\ln \left( \frac{r_s^2(t_s)}{A_s^2(f,t_s)} \right) = -2\pi f Q_s^{-1} t_{si} + \text{constant}. \quad (10)$$

We estimate $Q_s^{-1}$ values at each station by means of least-squares from the plots of the left-hand side against the travel time for many earthquakes. In this estimation, we take the spectral amplitude in six frequency bands having an octave width for the frequency range from 0.5 to 32 Hz. Finally $Q_s^{-1}$ values are estimated by taking the average of those values over stations in three subregions as illustrated in Figure 1: (a) northern Tohoku (N-Ohfunato), (b) southern Tohoku (S-Tohoku), and (c) Kanto.

Estimated $Q_s^{-1}$ values are shown in Figure 5 by filled symbols. The $Q_s^{-1}$ values in each subregion decrease with frequency according to the reciprocal of frequency. The values of $Q_s^{-1}$ are smaller in northern Tohoku and southern Tohoku than those in Kanto. In the following analysis, we use these $Q_s^{-1}$ values. In this figure, we also plot $Q_s^{-1}$ values reported in past studies for comparison: $Q_s^{-1}$ at Ohfunato (see Fig. 1) in northern Tohoku (Hoshiba, 1993), and those in southern Tohoku (Uetake and Ikeura, 2002). Although the gross features of frequency dependences are almost the same, the absolute values of $Q_s^{-1}$ in past studies are larger than our estimation. This is due to the differences in geometrical spreading factor; both of the past studies assume a constant velocity structure.
Values estimated at Ohfunato in northern Tohoku area by Hoshiba (1993); those estimated by Uetake and Ikeura (2002) in southern Tohoku area are also plotted.

Source Spectra

Using the obtained values of attenuation factors and site-amplification factors, we estimate source spectra by using equation (3). We show an example of the Fourier spectrum of seismic moment rate function $\bar{M}(f)$ in Figure 6. Figure 6a shows the epicenter (star) and the distribution of stations (dots) used for this example, and Figure 6b shows the estimated $\bar{M}(f)$. The earthquake magnitude and the focal depth of this event are 4.2 and 46.4 km, respectively. Thin gray curves in Figure 6b are spectra estimated by using one station’s record, and a black curve represents $\bar{M}(f)$ of this event, which corresponds to the rms average of the thin gray curves. By fitting equation (6) to $\bar{M}(f)$, we obtain the dashed curve in Figure 6b to estimate seismic moment $M_0$ and corner frequency $f_c$. Even though there are some small deviations, most of the obtained $\bar{M}(f)$ values are consistent with the omega square model.

Figure 6. An example of the source spectrum estimation. (a) Map of stations and the epicenter (star) of an earthquake with magnitude 4.2 and focal depth 46.4 km. (b) Fourier spectra of seismic-moment rate function $\bar{M}(f)$ (solid curves) with the correction of attenuation and site-amplification factors, where the thin gray curves are spectra for 11 station data. The bold broken curve is the best-fit omega-square model curve. (c) $\bar{M}(f)$ estimated without site-amplification factor corrections.
We have assumed $F_s = 2$ for all frequencies to take into account the reflected seismic waves from ground surface. The validity of this assumption depends on the overlying sedimentary layer’s effect to reflected waves. To ascertain the validity of this assumption, we compare the spectra estimated at two stations separated by 94 km: the reference site N.SNDH (1206 m in depth), which has a 250-m sedimentary layer overlying hard Triassic rock, and N.KKWH (120 m in depth, see Fig. 1), which has a thin sedimentary layer of a few meters in thickness overlying hard rock in Paleogene. We take the ratio of estimated spectra between two stations for 13 events observed. The average of this ratio is 1 for the whole frequency range, although ratios scatter within a factor of 2. This case study supports our assumption $F_s = 2$.

Figure 6c shows the Fourier spectrum of seismic-moment rate function $\dot{M}(f)$ when the correction of site factors are neglected, that is, $G_i(f) = 1$ for all stations. The spectra scatter much larger than those with site-factor correction shown in Figure 6b. When we neglect the site factor, the seismic moment is overestimated and the corner frequency is underestimated for all the cases as shown in this example.

Figure 7 shows a comparison of seismic moment $M_0$ estimated in this study with those published in the NIED centroid moment tensor (CMT) catalog based on the broadband data of the F-net (Fukuyama et al., 1998). Despite the short-period (0.5–32 Hz) data we used, we find fairly good agreement between them in a wide seismic moment range of $10^{14}$–$10^{17}$ N m.

Figure 8 shows the plot of seismic moment $M_0$ against corner frequency $f_c$. Brune’s stress drop $\Delta \sigma = (7/16)(M_0/(2.34\beta/2\pi f_c^3))$ (Brune, 1970) distributes in a range from 0.1 to 10 MPa. We also plot the measurements for earthquakes in Japan by Jin et al. (2000) and Kinoshita and Ohike (2002) in this figure. Our result shows a variation of Brune’s stress drop against seismic moment; $\Delta \sigma = 1$–10 MPa for $M_0 > 10^{14}$ N m, and 0.1–10 MPa for $M_0 < 10^{14}$ N m. The result of regression analysis is

$$\log M_0 = (-3.6 \pm 0.14) \log f_c + (16.6 \pm 0.10), \quad (11)$$

which is shown by a solid line in Figure 8. The conventional scaling law of $M_0 \propto f_c^{-3}$ (constant stress drop) does not fit our result. It is mainly due to small corner frequencies for small seismic-moment events. A strong power in the $M_0 - f_c$ relation was originally proposed by Iio (1986) for a wide seismic-moment range data. He concluded that the power becomes $-4$ for small seismic moments less than about $10^{16}$ N m. Recently, Jin et al. (2000) reported a steplike change in the $M_0 - f_c$ relation for shallow earthquakes. According to them, events of $M_0 > 10^{13}$ N m and $M_0 < 10^{11}$ N m obey the $M_0 \propto f_c^{-3}$ relation; however, the power becomes larger as $M_0 \propto f_c^{-(7/8)}$ for events between $10^{11}$ N m and $10^{13}$ N m. In their results, Brune’s stress drop exhibits a wide variation from 0.01 MPa to 10 MPa in $M_0 10^{11}$–$10^{13}$ N m.
creases with $M_0$ increasing. By fitting a line to the data, we get a regression relation:

$$\log \sigma_{ap} = (0.44 \pm 0.02) \log M_0 + (-0.89 \pm 0.30), \quad (12)$$

as is plotted by a black solid line in the figure.

Apparent stress in Figure 9 may be underestimated because $E_s$ is estimated from the source spectra in a limited frequency band of $f_{\text{min}} \le f \le 30$ Hz. Here we examine the contribution of “missing energy” (Ide and Beroza, 2001) from outside the frequency band of our analysis. Gray circles in Figure 9 represent the value of apparent stress after the correction of the missing energy in frequencies lower than $f_{\text{min}}$ and higher than 30 Hz estimated by using the extrapolation of the omega-square model. Apparent stresses with missing energy correction $\sigma_{ap}'$ become larger than those without correction, especially for smaller seismic-moment events. The regression line for corrected apparent stresses $\sigma_{ap}'$ is

$$\log \sigma_{ap}' = (0.39 \pm 0.02) \log M_0 + (0.13 \pm 0.33), \quad (13)$$

which is shown by a dashed line in Figure 9. The seismic-moment dependence is still seen.

The power of scale dependence decreases from 0.44 to 0.39 after the correction of missing energy. However, if we consider the $f_{\text{max}}$ in source spectra (Hanks, 1982), the correction may overestimate missing energy. From these results, we conclude that the apparent stress of small to moderate earthquakes in this region increases with seismic moment, and the power of scale dependence is between 0.44 and 0.39.

In Figure 10, we plot the apparent stress $\sigma_{ap}$ against the focal depth. To remove the seismic-moment dependence, the distribution is shown separately for three seismic-moment ranges. The apparent stresses (filled circles) in our result

Figure 9. Relation between apparent stress and seismic moment. Solid circles are results of the present study and gray circles are estimations including spectral contribution for frequency outside of $f_{\text{min}} \sim 30$ Hz by using the omega-square model. A solid line and a broken line are regression lines for solid and gray circles, respectively.

Figure 10. Plot of apparent stress against focal depth for three different ranges of seismic moment.
look independent of depth for all the seismic-moment ranges. Only a slight increase of apparent stress can be found for the depth from 30 to 50 km in small-moment events (Fig. 10a). The values in others works plotted by open symbols (Kinoshita and Ohike, 2002; Jin and Fukuyama, 2005) also indicate a slight increase of apparent stress for the same depth range for larger earthquakes (Fig. 10b, c). However, the scatter of data is too large to discuss the depth dependence in detail.

Discussion

The scale dependence of apparent stress is clearly observed in our analysis by using adequate site and attenuation factors. Contrary to the speculation of Ide and Beroza (2001), the scale dependence still remains even after the correction of site factors and missing energy. They also argued another possible origin of artificial scale dependence of apparent stress: missing events having larger corner frequencies because of the upper bound of the frequency band, 30 Hz in our case. Figure 8 indicates that we cannot completely deny such a possibility for the seismic moment range less than $10^{13}$ Nm. But the apparent stress in a seismic-moment range from $10^{13}$ to $10^{17}$ Nm, which will not contain such missing events, clearly increases with the seismic moment increasing. Thus, we conclude that the apparent stress shows scale dependence at least in this moment range.

If the source spectra obey the omega-square model and $M_0$ is proportional to $f_c^{-3}$, that is, Brune’s stress drop is scale independent and apparent stress also should be scale independent (Kanamori and Rivera, 2004). Then $M_0 \propto f_c^{-3/4}$, we have the relation

$$\sigma_{ap} = \frac{2\pi\mu}{\rho p^2} (R_{1zh})^2 \frac{\pi}{4} M_0 f_c^{-3} \propto M_0^{3/4}. \quad (14)$$

by using equations (4)–(6) and integrating the squared velocity spectra from 0 to infinity in frequency. In the case of $n = 3.6$ as given by equation (11), the power of $M_0$ becomes 0.17. If the limited frequency band is considered, the power of $M_0$ becomes about 0.2 for $10^{11}$ Nm $< M_0 < 10^{17}$ Nm. The predicted power of 0.17–0.2 is smaller than our estimated value 0.39–0.44; however, apparent stress becomes scale dependent. If seismic moment $M_0$ is chosen as an independent variable for the regression analysis of $M_0 - f_c$ relation for data plotted in Figure 8, the frequency dependence of seismic moment becomes stronger as $M_0 \propto f_c^{-4.8}$, which results in $\sigma_{ap} \propto M_0^{3/8}$ by using equation (14). The limited frequency-band correction increases the power to 0.41, which explains our estimated value of 0.39–0.44. It will be possible to expect that slight deviations of source spectra from the best-fit omega-square model might affect the power of scale dependence of apparent stress, but we may say that some of the scale dependence of apparent stress is attributed to the scale dependence of Brune’s stress drop.

The depth dependence of apparent stress is not clear in our result. However, the temperature and confining stress, which strongly depend on depth, may affect the fault property and rupture process. Analyzing aftershocks of the 1994 Northridge, California, earthquakes, Mori et al. (2003) reported a small depth dependence of apparent stress. According to the result in Jin and Fukuyama (2005), the apparent stresses of shallow earthquakes in volcanic areas (large heat-flow areas) are significantly small as compared with other shallow earthquakes in southwestern Japan. Those small apparent stresses are in the range of $10^4$–$10^5$ Pa for seismic moment of about $10^{16}$ Nm, which are one to two orders smaller than those in nonvolcanic areas. Because the geologic conditions between the volcanic area and subduction zone in our analysis are significantly different, it may be difficult to distinguish such temperature effect in our result.

Large scatter of apparent stress in Figure 10 may contain not only statistical variation but also its spatial variation as pointed out by Kinoshita and Ohike (2002). They found spatial variations in source spectra and the power of scale dependence of apparent stress for earthquakes in the Philippine Sea slab beneath the Kanto Region. In the present study, however, it is difficult to resolve those effects because of the sparse distribution of events used for the analysis. We need further careful investigation with a dense data set to reveal the depth dependence and regionality of apparent stress.

Conclusion

In this study, we carefully measured apparent stress of small to moderate earthquakes occurring along the subducting Pacific plate in northeastern Honshu, Japan. Site-amplification factors and attenuation factors are estimated with the coda normalization method for the frequency range from 0.5 to 32 Hz. In $Q_s^{-1}$ estimation, we take the effect of geometrical spreading factor in the 1D velocity structure into consideration. The obtained site-amplification factors relative to that at a deep-borehole site show a clear frequency dependence, which is strongly related to the geologic age and lithology of rock. At young sedimentary rock sites, the site-amplification factor is 2–30 for the 0.5–1.0 Hz band, and 0.2–1.0 for the 16–32 Hz band. Estimated $Q_s^{-1}$ values vary in proportion to the reciprocal of frequency and show a little regional variation: $Q_s^{-1} \sim 10^{-2}$ for 0.5–1.0 Hz and $Q_s^{-1} \sim 10^{-3}$ for 8–16 Hz. The correction by using the site-amplification and attenuation factors remarkably reduces the scatter of source spectra. Most of the corrected source spectra show a good agreement with the omega-square model up to 30 Hz, although some of them show a small deviation from the omega-square model.

The estimated apparent stress increases from $10^4$ to $10^7$ Pa with seismic moment increasing from $10^{13}$ to $10^{17}$ Nm. This scale dependence is not an artificial result caused by the limitation of the frequency band. The power of scale dependence is estimated to be 0.39–0.44. We may say that some of the scale dependence of apparent stress is attributed
to the scale dependence of Brune’s stress drop. The depth dependence of apparent stress is not clear in our result, although a slight increase with depth is suggested for the depths of 30–50 km.

Acknowledgments
We thank the associate editor and three anonymous reviewers. Their comments have made significant improvement to this manuscript. We sincerely thank Jin Anshu and Katsuhiro Shiomi for giving us helpful information to analyze the Hi-net data. This work is partially supported by the Grant-in-Aid for Japan Society for the Promotion of Science Fellows (No. 10530), and the sponsorship of the Japan Nuclear Energy Safety Organization open-application project for enhancing the basis of nuclear safety, and the Earthquake Research Institute, University of Tokyo, cooperative research program. This work was conducted as a part of the 21COE program on Earth Science of Tohoku University. Seismic Analysis Code (SAC) and Generic Mapping Tools (GMT) code are used for data processing and figure plotting.

References
Saito, T., H. Sato, and M. Ohtake (2002). Envelope broadening of spheri-
cally outgoing waves in three dimensional random media having
inhomogeneous lithosphere based on the parabolic approximation:
Southeastern Honshu, Japan, J. Geophys. Res. 94, 17,735–17,747.
Sugimura, A. (1960). Zonal arrangement of some geophysical and petro-
logical features in Japan and its environs, J. Fac. Sci. Univ. Tokyo,
Sect. 2 12, 133–153.
of S-waves from strong motion records in spatial relation to surface
Tsujiura, M. (1978). Spectral analysis of the coda waves from local earth-
Uetake, T., and T. Ikeura (2002). Site amplification factors of K-NET sites
in Pacific coast region of southern Tohoku district, Japan, J. Seism.
Umino, N., and A. Hasegawa (1984). Three-Dimensional Qs Structure in
the Northeastern Japan Arc, J. Seism. Soc. Jpn. Ser. 2 37, 217–228

Yoshimoto, K., H. Sato, and M. Ohtake (1993). Frequency dependent at-
tenuation of P and S waves in the Kanto area, Japan based on the

Department of Geophysics
Graduate School of Science
Tohoku University
Aramaki-aza Aoba 6-3, Aoba-ku, Sendai
980-8578 Japan
(T.T., H.S., M.O.)

National Research Institute for Earth Science and Disaster Prevention
Ten’nodai 3-1, Tsukuba
305-0006, Japan
(K.O.)

Manuscript received 13 April 2004.