Power-law decay characteristic of coda envelopes revealed from the analysis of regional earthquakes

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[1] Seismic envelope of coda waves is an efficient measure to estimate physical parameters of the Earth heterogeneity. For NS component seismograms of regional earthquakes recorded by 11 IRIS stations, applying the Hilbert transform to make seismic envelopes in period bands from 0.25–0.5s to 16–32s with lapse time as long as 4000s, we discovered that coda envelopes decay according to some power of lapse time measured from the origin time. It is new and different from the present knowledge about coda envelopes which have been described by several bending curves with different decay rates. The ‘power-law’ decay characteristic of seismic envelopes indicates that the spectra amplitude of seismic coda could be simply expressed as of the form

\[ A(t, T_C) \propto t^{-\alpha(T_C)} \]

where \( t \) is lapse time and \( T_C \) is central period in second. Applying regression analysis, we found that the exponent \( \alpha \) value has period dependence and ranges \( \alpha_1 = 1.3\text{–}6.5 (0.25\text{–}0.5s \text{–}16\text{–}32s), \alpha_2 = 0.6\text{–}6.1 (0.5\text{–}1s \text{–}16\text{–}32s) \) for before and after ScS arrival, respectively. Moreover, it shows no significant difference between shallow and deep focus events for periods shorter than 8s. The simple and distinct characteristics of seismic coda envelopes could provide reliable information to determine physical characteristics such as attenuation and scattering to identify the regional difference of medium heterogeneity in the Earth. Citation: Lee, W. S., and H. Sato (2006), Power-law decay characteristic of coda envelopes revealed from the analysis of regional earthquakes, Geophys. Res. Lett., 33, L07317, doi:10.1029/2006GL025840.

1. Introduction

[2] Coda waves of regional earthquakes are the most prominent evidence of the Earth medium heterogeneity. Their spectral amplitude and envelope decay gradient reveal the spatial distribution of heterogeneities and attenuation. Rautian and Khalturin [1978] studied the amplitude of coda waves for a wide range of lapse times and period bands in central Asia, which was the pioneer work to describe the characteristics of coda envelopes using analog instruments. They reported that early portions of the coda amplitudes are different from station to station; however, band-pass filtered traces have a common shape at all stations after about two times the S-wave travel time from the source to the receiver. Furthermore, they showed that the coda envelope in each frequency band has a few lapse time segments. Since the 1980s, there have been a lot of investigations for estimating attenuation parameters, for example, coda \( Q \), from the stable characteristics of coda waves based on scattering models worldwide.

[3] To date, seismic coda envelopes have been described as the following conventional form,

\[ A(t, T_C) \propto t^{-\alpha} \exp(-\pi t Q_C T_C) \]

where \( A \), \( t \) and \( T_C \) are amplitude, lapse time and central period, respectively, and the power \( n \) depends on a geometrical spreading [Aki and Chouet, 1975]. Multiplying a geometrical spreading factor to the LHS and taking logarithm, we can estimate a parameter \( Q_C \), called coda \( Q \), by using a linear regression analysis. Practically, the shape of seismic envelopes is well presented on a semi-logarithmic scale. For instance, the features of the change of coda decay gradient and offset around ScS arrival clearly appear when we plot coda envelopes using the scale [Lee et al., 2003, 2006]. Because of these advantages, most of time domain analyses for coda waves have been expressed by using semi-logarithmic plots.

[4] In this study, having the necessity of bringing the classic work by Rautian and Khalturin [1978] up to date, we newly summarize the phenomenological features of seismic coda envelope decay investigating regional earthquakes for various regions and wide period bands as long as 4000s in lapse time so that the characteristics of attenuation in the deep Earth could be elucidated. We will also show that coda envelope decay would be followed not by the conventional expression but by ‘power-law’ on a log-log plot. Since a log-log plot is rather applicable to demonstrate seismic envelopes for a long lapse time range visually, we plot coda envelopes on a log-log scale instead of a semi-logarithmic scale.

2. Data

[5] Broadband NS-component velocity seismograms of regional earthquakes recorded by 11 IRIS seismic network stations distributed over the world were used in this study. In order to compare the characteristics between shallow and deep focus events and to examine seismic coda envelopes globally, we collected both the earthquakes occurred at shallower than 50 km (118 events) and deeper than 150 km (39 events) in depth during the period from 1988 to 2005 with epicentral distances less than 10° from individual stations (Figure 1). Their moment magnitudes \( (M_b) \) range from 4.7 to 7.8.

3. Seismic Envelopes

[6] Heterogeneities randomly distributed in the Earth work as generators of incoherent seismic waves. Coda waves are interpreted to be composed of those scattered waves. In general, we often disregard their phase information and focus on their smooth amplitude envelopes such as root mean square (RMS) envelopes, since the concept of energy transportation is rather applicable approach than that...
of wave propagation to model randomly inhomogeneous media. [Sato and Fehler, 1998].

[7] We made band-pass filtered seismograms with applying fourth-order Butterworth filters which have pass bands of 0.25–0.5, 0.5–1, 1–2, 2–4, 4–8, 8–16 and 16–32s in this study. Instead of applying the RMS technique which executes smoothing procedure, we calculate the envelope function by the Hilbert transform for individual band-pass filtered seismograms to preserve phase information.

[8] Figure 2 shows a band-pass filtered trace (gray) having a 16–32s pass-band occurred at central Asia and its envelope function (black). ScS and sScS phases are clearly shown around 850 and 1000s in lapse time, respectively. After making seismic envelopes, we normalized the average coda level at specific lapse time window considering each period band (e.g., lapse time ranging 500–600s for 1–2s band, and 650–750s for longer than 2–4s period bands) to correct the source-size differences. The lapse time window of coda normalization is represented by bold black bars in Figure 3. At periods shorter than 1s, we choose 50s lapse time window after 1.5 times average S-wave travel time which is not marked in the figure for each station since coda waves are rapidly attenuated. To characterize the general features of coda envelope, we carry out envelope stacking to normalized seismogram envelopes calculated by 10–15 earthquake events for individual stations and period bands. We stacked together seismogram envelopes obtained at PAS and SBC stations (California, USA) separated about 1.5° which is small enough compared to scattering volume due to a small number of good signal-to-noise ratio data.

[9] Examining seismic coda envelopes, for various regions and wide period bands up to 4000s in lapse time, we found that the decay gradient of coda envelopes with lapse time shows ‘straight-line’ on a log-log scale plot (Figure 3). That is, seismic coda amplitude is governed by ‘power-law’ decay instead of the conventional description on later coda. Moreover, in Figure 3, we can find that the coda decay gradient at shorter periods is steeper than that at longer periods for both shallow and deep focus events.

4. Power-Law Decay and Frequency Dependence

[10] We summarize stacked seismic envelopes of regional earthquakes for 7 period bands in Figure 3. Upper and lower panels represent the envelopes for shallower than 50 km and deeper than 150 km in focal depth, respectively. Station codes are shown at the right-most part of each panel. The theoretical ScS arrival calculated by the *iasp91* model [Kennett and Engdahl, 1991] is marked by a vertical gray bar. Confirming ‘power-law’ feature for all stations and period bands, we perform regression analysis to determine the best-fitting line which minimizes the errors between the observed coda amplitude and the line on a log-log scale for individual period bands. We set a form of regression line as, \[ A(t, f_c) = C \cdot t^{-\alpha} \] where \( C \) is a constant. Accordingly, we can estimate the exponent of lapse time, \( \alpha \), for all stations and period bands. Lapse time window for regression analysis is decided after about twice of S-wave arrival time or later to select stable coda waves [Rautian and Khalturin, 1978], and to avoid being severely contaminated by surface waves. The best-fitted lines are marked by straight lines in Figure 3. \( \alpha_1 \) and \( \alpha_2 \) range 1.3–6.5 for before ScS at 0.25–0.5–16–32s periods and 0.6–6.1 for after ScS at 0.5–1–16–32s periods, respectively. Investigating \( \alpha \) values worldwide, we found regional difference of \( \alpha \) such as slower decreasing (low \( \alpha \)) at TATO (Taiwan). The slow decay of coda amplitude might be caused by a weak absorption or strong scattering beneath the station. Multiple coda decay branches are also observed at LPAZ station in this study, which is another good example to show regional difference of scattering and attenuation structure in the deep Earth.

[11] In general, the decay rate of coda is steeper from the arrival of ScS except for the data of AAK (Kyrgyzstanz) and LPAZ (Bolivia) in Figure 3. Offset of coda amplitude associated with ScS arrival appears in ISP (Turkey), AAK and LPAZ at 4–8, 8–16 and 16–32s period bands as well, which is well explained by the attenuation mechanism of much stronger scattering loss than intrinsic absorption in the lower mantle though the scattering energy is negligibly small [Lee et al., 2003, 2006].

[12] The systematic change of coda decay rate which Rautian and Khalturin [1978] reported could not be found even at AAK, which is located in central Asia near Garm region, except for once occurring around ScS arrival and several decay gradient branches at LPAZ in this regional earthquake study. It is quite natural since they patched together envelope segments having narrow dynamic ranges collected by local and regional events to construct individual whole envelope curves with wide dynamic ranges up to 160 dB for various frequency bands as long as 10,000s in lapse time different from this study.

[13] Typically for the regional earthquake study, it is hard to collect sufficient data which meet our criteria (weak...
influence of surface waves, larger than moment magnitude 4.5 with epicentral distances less than 10°C176/C176, and good S/N ratio) in continents. The criteria are appropriate for examining coda envelopes as long as 4000s in lapse time. Such data what we need are mainly obtained in the vicinity of the plate boundaries. Consequently, most of our data involve a large amount of oceanic crust, which has a great advantage to efficiently suppress Lg coda in particular, except for the case in central Asia. Yet wherever we investigate coda envelopes in this study, the feature of power-law appears even in a continent. In order to discuss whether the power-law decay is region-specific, it would be necessary to examine coda envelopes for local earthquakes as well over the world.

The phenomenological feature of frequency dependence for seismic coda decay gradient at long lapse time has been reported by investigating regional earthquakes [Lee et al., 2003, 2006]. In this study, we show strong period dependence of α value by performing multiple nonlinear regression analysis introducing the regression form as $\alpha(T_C) = a \log_{10} T_C + b$, where $a$ and $b$ are constants, and $T_C$ is central period in second. Figure 4 presents resultant plots for $\alpha$ values corresponding to central periods. From the regression for both shallow and deep events, we determine $a$ and $b$ values as the following: $a_1 = -2.0(\pm 0.4)$, $b_1 = 4.0(\pm 0.2)$, and $a_2 = -2.5(\pm 0.5)$, $b_2 = 4.8(\pm 0.5)$, where the subscripts 1 and 2 denote, respectively, before and after ScS arrival. To compare the coda envelope characteristics between shallow and deep events, we took the same analysis for both cases separately. Following are the evaluated expressions; $\alpha_{sh} = -2.2(\pm 0.4) \log_{10} T_C + 4.2(\pm 0.2)$, $\alpha_{sh} = -1.4(\pm 0.7) \log_{10} T_C + 3.6(\pm 0.4)$, and $\alpha_{dp} = -2.2(\pm 0.7) \log_{10} T_C + 4.7(\pm 0.7)$, $\alpha_{dp} = -3.0(\pm 0.6) \log_{10} T_C + 4.9(\pm 0.5)$, where the subscripts are the same as previous, and the superscripts sh and dp represent for shallow and deep events, respectively. According to these expressions, we can find that the difference between $\alpha_{sh}$ and $\alpha_{dp}$ is not quite significant. One thing we must note, in Figure 4(left), is that the influence of shallow focus events on the regression result seems to be stronger than that of deep focus events due to larger number of data. It would be overcome when we analyze much more data for deep events in near future. However, resultant values may not be considerably different from this result.

Consequently, we suggest that the spectra amplitude of seismic coda takes the following simple expression, $A(t, T_C) \propto t^{-\alpha(T_C)}$. (1)

5. Discussion

Most of the researchers who found such a lapse-time dependence of coda decay rate have suggested that the later portion of coda is mainly dominated by the energy that has propagated in lower attenuation zones [e.g., Roecker et al., 1982]. Developing a single isotropic-scattering model for
the case where total scattering coefficient, $g_0$, decreases according to an inverse power of depth, Gusev [1995] reported that the theoretically predicted coda attenuation $Q_c^{-1}$ decreases with increasing lapse time. That is, seismic envelopes should be described by several $Q_c^{-1}$ values even by different geometrical spreading factors, and the attenuation model assumed the uniform distribution of scatterers in heterogeneous media is not suitable for explaining the lapse-time dependence of the coda decay rate.

[17] The ‘power-law’ decay with lapse time on later coda is new and different from reported coda envelope decay characteristics, and it leads us to consider non-uniform distribution of scatterers such as depth-dependent [e.g., Hoshiba, 1994] or fractal [e.g., Sato, 1988] in the Earth media since we hardly understand the feature with just regarding exponentially attenuated coda amplitude. A new attenuation parameter $\alpha$ value which contains both attenuation property and the information of scattering characteristic in heterogeneous media makes us possible to explain the decay gradient of seismic coda envelopes with only one parameter. The simple and distinct characteristic enables us to rather easily apply it to find out physical characteristics such as attenuation or source size. Thus, we note that it is important to carefully establish a theoretical model for these phenomena and precisely figure out the physical meaning of $\alpha$ value in near future.

[18] As pointed out in section 4, much steeper decay rate after $ScS$ is observed, generally. The difference of decay rate around $ScS$ provides us crucial information to make a numerical scattering model based on the radiative transfer theory for imaging the heterogeneous Earth medium much accurately. In this sense, when one seeks to use S-coda envelopes to investigate heterogeneity in the lower mantle, it is appropriate for selecting the period band at least longer than 2s (see Figure 3). Besides, we showed that there is no significant difference between shallow ($<50$ km) and deep ($>150$ km) focus events for $\alpha$ values. It is presumed to be related to the fact that the influence of surface wave scattering is not so serious before $ScS$ at $0.25$–$0.5s$–$4$–$8s$ period bands, and scattered waves of the $ScS$ waves sometimes dominate over scattered $S$ waves as well as scattered surface waves after $ScS$ [Lee et al., 2003] at $2$–$4s$–$16$–$32s$ period bands, respectively. Therefore, we are able to use both data to examine the medium heterogeneity in the deep Earth so that it enables us to extend the spatial coverage of our study region.

6. Conclusions

[19] Summarizing the phenomenological features of coda envelope for wide period bands with long lapse time range, we showed coda envelopes can be described by ‘power-law’, $A(t,T_c) \propto t^{-\alpha(T_c)}$, and $\alpha$ value has strong period dependence. The simple and distinct characteristics of seismic coda envelopes could provide reliable information to determine physical characteristics such as attenuation, and to identify the regional difference of medium heterogeneity in the deep Earth. There is no significant difference of coda decay rate between shallow ($<50$ km) and deep ($>150$ km) focus events for shorter than 8s in period, and no systematic change of coda decay rate with lapse time was found except for once occurring around $ScS$ arrival and the case of LPAZ (multiple changes).

[20] We will have to examine the validity of the ‘power-law’ decay for the study of local earthquakes, and the $\alpha$ value should be studied in correlation with tectonic structure in the Earth medium as well.

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References


