Synthesis of vector-wave envelopes in 3-D random media characterized by a nonisotropic Gaussian ACF based on the Markov approximation

Haruo Sato

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[1] The earthquake source duration is short; however, the apparent duration time of observed seismogram increases with travel distance. The amplitude excitation is observed even on the transverse component for P waves, on the longitudinal component for S waves. These phenomena are well explained by scattering due to random velocity inhomogeneities around the global seismic ray. We directly synthesize vector-wave envelopes in 3-D random elastic media statistically characterized by a nonisotropic Gaussian autocorrelation function (ACF). The method uses the Markov approximation in the case that the wavelength is shorter than the correlation distance and the ray direction is parallel to one of the principal axes of the ACF. A spherical outgoing vector wavelet radiated from a point source in the random elastic media is used as a basic model for high-frequency seismogram envelopes from micro-earthquakes in the inhomogeneous lithosphere. The stochastic master equation for the two-frequency mutual coherence function (TFMCF) of the potential field is analytically solved. The Fourier transform of TFMCF gives mean square (MS) envelopes of band-pass filtered vector-wave traces. If the ACF is axially symmetric around the ray direction, MS envelopes of vector components are analytically solved. The aspect ratio of the correlation distance in the longitudinal direction to that in the transverse direction is the key parameter for the envelope broadening and the excitation in the orthogonal component. Envelope broadening becomes longer and the transverse (longitudinal) component amplitude increases for a P wavelet (for an S wavelet) when the correlation distance in the transverse plane becomes smaller. When the vertical correlation distance is shorter than the horizontal one as seen in the real Earth, the envelope broadening is larger for horizontal raypaths compared with vertical raypaths.


1. Introduction

[2] In high-frequency vector-wave seismograms, the apparent duration time is much longer than the earthquake source duration time because of scattering from random inhomogeneities in the earth medium. Envelope broadening with increasing travel distance is clearly seen in S wave seismograms from local micro-earthquakes [Sato, 1989; Petukhin and Gusev, 2002]. Envelope broadening is well explained by scattering due to random velocity inhomogeneities in a narrow angle along the forward direction on the basis of the Markov approximation when the wavelength is shorter than the correlation distance [Sato, 1989; Fehler et al., 2000; Saito et al., 2002]. The Markov approximation, which is a stochastic extension of the phase screen method, predicts well the envelope of a wavelet propagating through random media [Shishov, 1974; Sreenivasiah et al., 1976; Lee and Jokipii, 1975; Ishimaru, 1978; Rylov et al., 1989; Sato and Fehler, 1998; Saito et al., 2007; Takahashi et al., 2008]. S wave envelope broadening is found to be small in the fore arc side and large in the back arc side of the volcanic front along the Japan arc [Obara and Sato, 1995]. Envelope broadening is large for seismic raypaths traveling beneath Quaternary volcanoes compared with those for raypaths traveling between Quaternary volcanoes in northern Japan [Takahashi et al., 2006]. These documented differences in the envelope broadening reflect regional variations in the spectrum of medium inhomogeneity. For teleseismic P waves, there is excitation of the transverse component caused by scattering in the lithosphere. Kubanza et al. [2006, 2007] reported that the energy partition of P and P coda waves onto the transverse component is large at sites located in plate boundaries but small in stable continents. Both the envelope broadening of the vertical component and the energy partition onto the transverse component can also be explained by scattering of vector waves in random elastic media. There have been developments in the Markov approximation.
Nonisotropic Random Media

Figure 1. Example of an axially symmetric nonisotropic random medium in an infinite 3-D space and a point source. Rays V and H are parallel to the short and long correlation distance directions, respectively.

approximation for the envelope synthesis of a vector wavelet (Korn and Sato [2005], Sato [2006] (hereinafter referred to as Paper I), Sato [2007] (hereinafter referred to as Paper II), and Sato and Korn [2007]); however, these theories are restricted to random media characterized by isotropic autocorrelation functions (ACFs).

There are evidences for anisotropy found in geological structures and well log data. Holliger and Levander [1992] examined geological maps and rock properties in the Ivrea zone, northern Italy that is believed to be an exposed section of the lower crust. Digitizing the lithological map, they found that the ACF is a von Kármán type with nonisotropic randomness: the shortest correlation distance is 150—180 m and the longest one is 550—750 m, producing an aspect ratio of 3—5. In another study, the anisotropy was estimated from the analysis of log data of closely spaced wells at the KTB deep boreholes in Germany [Wu et al., 1994]; the characteristic length in the horizontal direction is 1.8 times larger than that in the vertical direction.

There are seismological studies that document the existence of nonisotropic random inhomogeneities. Nielsen and Thybo [2003] analyzed teleseismic Pn arrivals with extensive codas observed to offsets beyond 3000 km from the peaceful nuclear explosion seismic profiles in the western part of the former Soviet Union. At high frequencies (5—10 Hz), the scattered wave trains extend for more than 10 s; however, at low frequencies (0—2.5 Hz) the wave trains are only 3 s long. The authors suggest that these teleseismic Pn waves travel as an upper mantle whispering gallery phase and that the origin of the long coda is crustal scattering on the basis of numerical simulations. In their model the lower crustal random inhomogeneities from 15 to 40 km depth are characterized by a von Kármán type ACF with a Hurst number of 0.3, and horizontal and vertical correlation distances are 2.4 and 0.6 km, respectively.

Furumura and Kennett [2005] proposed a scattering slab model as an efficient waveguide to explain earthquake intensity anomalies observed in Japan. The slab is modeled as a quasi-laminated random velocity structure with 3% in RMS fractional velocity fluctuation and short correlation distance (0.5 km) across the plate thickness and longer correlation distance (10 km) along the dipping direction. Their model predicts that high-frequency ($f > 2$ Hz) signals are well guided and low-frequency ($f < 0.25$ Hz) signals travel as a forerunner.

These studies motivate the quantification of vector-wavelet as it travels through nonisotropic random media. Using numerical simulations, Hong and Wu [2005] found that the level of scattering is not very sensitive to the scale variation in the wave incidence direction, but is highly sensitive to the scale variation in the transverse direction. Margerin [2006] synthesized scalar wave envelopes in nonisotropic random media on the basis of radiative transfer theory with a directional scattering coefficient focusing on the coda portion. Saito [2006] simulated the scalar wave envelope in nonisotropic 2-D random media on the basis of the Markov approximation. His synthesis shows that envelopes increase in duration and decrease in maximum amplitude more rapidly for propagation parallel to the longer correlation distance direction than for propagation parallel to the shorter correlation distance direction.

Here, we present the three-component envelope synthesis of a vector wavelet in nonisotropic random media for impulsive radiation from a point source on the basis of the Markov approximation. For mathematical simplicity we use a nonisotropic Gaussian ACF for characterizing the random media. Focusing on a P wavelet, we investigate the sensitivity to the ray direction and the orientation of the non-isotropy of randomness. The three-component envelope synthesis for the incidence of a plane wavelet onto a random medium is given in the Appendix.

2. Vector-Wavelet Envelopes in Randomly Inhomogeneous Media

We study the propagation of a wavelet radiated from a point source in nonisotropic random elastic media in 3-D infinite space with a statistical method. A cartoon of a nonisotropic random medium is shown in Figure 1. Random media are statistically characterized by a Gaussian ACF with three correlation distances. If the medium inhomogeneity is weak and the dominant wavelength is much smaller than any of the correlation distances, there is little conversion scattering between $P$ and $S$ waves. In this case the propagation of $P$ wave and $S$ wave are independent, and each wave is represented by its potential. We develop the three-component envelope synthesis of a vector wavelet along a global ray that is parallel to one of the principal axes of the nonisotropic ACF.

2.1. $P$ Wavelet Case

2.1.1. Parabolic Equation for Spherically Outgoing Wave

We first study the propagation of $P$ wavelet radiated from a point source at the origin in a randomly inhomoge-
neous elastic medium, where the $P$ wave velocity $\alpha(x) = V_0(1 + \xi(x))$ has a small fractional fluctuation $\xi(x)$ around the average velocity $V_0$: $|\xi| \ll 1$. In the following, we develop mathematics according to Paper II to avoid duplication. The $P$ wave displacement vector is written at $u = \nabla \phi$ by using the scalar potential $\phi$, which is isotropic at the source. We write the scalar potential as a superposition of spherically outgoing harmonic waves of angular frequency $\omega$, $\phi = (1/2\pi) \int_{-\infty}^{\infty} (U(ik_0) e^{ik_0r-\omega t} dr$, where $k_0 \equiv k/\sqrt{V_0}$ is the wave number and the distance $r$ from the source is much larger than the wavelength. We focus on the wave propagation with the global ray direction along the $z$ axis. Since the wavelength is shorter than any of the correlation distances, $U$ is governed by a parabolic equation,

$$2ik_0\partial_r U + \Delta_1 U - 2k_0^2 \xi U = 0,$$

where $\Delta_1 = \partial_x^2 + \partial_y^2$ is the Laplacian for local coordinates $x_1 = (x, y)$ on the transverse plane at a distance $r$ along the $z$ axis from the source.

2.1.2. Ensemble of Random Media and the Markov Approximation

[9] We imagine an ensemble of random media. The fractional fluctuation $\xi(x)$ is assumed to be a statistically random function of space coordinate $x$ with $\langle \xi(x) \rangle = 0$, and angular brackets represent the average over the ensemble. The ensemble is statistically characterized by the ACF: $R(x) \equiv \langle \xi(x) \xi(x' + x) \rangle$, which is statistically homogeneous but nonisotropic. Here, we use a nonisotropic Gaussian ACF,

$$R(x) = \varepsilon^2 e^{\frac{x_1^2}{k_0^2} + \frac{x_1^2}{k_0^2}},$$

which is parameterized by a mean square (MS) fractional fluctuation $\varepsilon^2$ and three correlation distances, $a_x, a_y$, and $a_z$.

[10] The two-frequency mutual coherence function (TFMCF) is defined as a correlation of $U$ between different two different locations $(x_1', x_1')$ on the transverse plane at a distance $r$ along the $z$ axis in the vicinity of $\theta = 0$ and different angular frequencies $\omega$ and $\omega'$, $\Gamma_2(x_1, r, \omega, \omega') \equiv \langle U(x_1', r, \omega') U(x_1', r, \omega') \rangle$ [see Ishimaru, 1978]. It is written by using the difference coordinate $x_{1,ld} = x_1' - x_1'$, and the center of mass angular frequency $\omega_c = (\omega + \omega')/2$ and the difference angular frequency $\omega_d = \omega' - \omega$. The TFMCF is independent of the center of mass coordinate $x_{1,c} = (x_1' + x_1')/2$ near the $z$ axis because of the homogeneity of the randomness and the isotropic source radiation.

2.1.3. Stochastic Master Equation for Quasi-Monochromatic Waves

[11] Using causality and neglecting backscattering, we derive the stochastic master equation for TFMCF for quasi-monochromatic waves ($\omega_d \ll \omega_c$) on the basis of the Markov approximation under the condition that the MS phase fluctuation is small $\varepsilon^2 a_x^2 k_0^2 \ll 1$ [Rytov et al., 1989, p. 108]. Factorizing the TFMCF as $\Gamma_2 = \partial_0 \Gamma_2 \cdot \tilde{w}(r, \omega_d)$, where $\tilde{w}(r, \omega_d) \equiv e^{-i\omega_d t/2}$ is the wandering term, we have the master equation for $\partial_0 \Gamma_2$:

$$\partial_0 \Gamma_2 + \frac{k_0^2}{2k_0^2} \Delta_1 \partial_0 \Gamma_2 + k_0^2 [A(0) - A(r, \omega_d)] \partial_0 \Gamma_2 = 0,$$

where $A$ is the longitudinal integral of ACF along the $z$ axis [Lee and Jokipii, 1975]. At long travel distances, contribution of this function at a small transverse distance becomes dominant. In the case of a Gaussian ACF, we have

$$A(x) \equiv \int_{-\infty}^{\infty} dz R(x, z) \approx \sqrt{\pi} e^{a_x^2/2} \left[ 1 - \frac{x_1^2}{a_x^2} - \frac{x_2^2}{a_y^2} \right]$$

for $|x| \ll a_x$ and $|y| \ll a_y$.

2.1.4. Intensity Spectral Density

[12] The wave intensity of each vector component is defined as the ensemble average of the square of displacement $I_j^p \equiv \langle |u_j|^2 \rangle = (\partial_j \phi \partial_i \phi^* \partial_j \phi \partial_i \phi^*)$. Taking the same procedure as given by Paper II, where the differentiation is calculated by using the local Cartesian coordinates, we define the intensity spectral density (ISD) of each component, which means the MS envelope of the band-pass filtered trace with central angular frequency at $\omega_c$:

$$I_j(r, t, \omega_c) = \frac{1}{2\pi^2} \int_{-\infty}^{\infty} d\omega_d e^{-i\omega_d t} \tilde{w}(r, \omega_d)$$

and

$$I_j^p(r, t, \omega_c) = \frac{1}{2\pi^2} \int_{-\infty}^{\infty} d\omega_d e^{-i\omega_d t} \tilde{w}(r, \omega_d)$$

where the reference ISD is defined as

$$I_j(r, t, \omega_c) = \frac{1}{2\pi^2} \int_{-\infty}^{\infty} d\omega_d e^{-i\omega_d t} \tilde{w}(r, \omega_d)$$

2.1.5. Wandering Effect

[14] The Fourier transform of the factor $\tilde{w}(r, \omega_d)$ with respect to $\omega_d$ gives the wandering effect at a distance $r$ in the time domain,

$$w(r, t) \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega_d \tilde{w}(r, \omega_d) e^{-i\omega_d t}$$

$$= \frac{V_0}{\sqrt{2\pi \sqrt{\pi^2 a_x^2}}} \left[ \frac{1}{\sqrt{\pi}} (t + \tau) \right]^2.$$
This factor is called the wandering term since the above equation shows the Gaussian distribution of travel times of wavelets at the receiver caused by the long wavelength components of the random velocity inhomogeneity; however, it does not influence the broadening of individual wave packets. Taking \( \tilde{w} = 1 \) in (5a), (5b), (5c), and (6), we define ISDs without the wandering effect as \( \Gamma_L^0 \), \( \Gamma_L^0(0, \tau, r) \), and \( \Gamma_L^0 \), where subscript “0” means no wandering effect. For the evaluation of complete ISDs we calculate the convolution of the wandering term and ISDs without the wandering effect in the time domain or through multiplication in the angular frequency domain. As shown later, the envelope broadening effect caused by scattering is proportional to the square of distance, which dominates over the wandering effect as the distance increases since the standard deviation in (7) is proportional to the square root of the distance.

2.1.6. Initial Condition

For impulsive isotropic radiation of the scalar potential from a point source at the origin, the initial condition is

\[
o\Gamma_2(x, t; r = 0, \omega_x, \omega_y) = 1/(4\pi).
\]

The corresponding ISD at the origin is \( \hat{I}^0(r, t, \omega) = \hat{I}^0(0, t, \omega) = 0 \) and \( \hat{I}^0(0, t, \omega) = (1/4\pi\tau^2)\delta(t - r/V_0) \) along the z axis. The ISDs given by (5a), (5b), and (5c) are propagators for a delta function source. In order to evaluate the MS envelope of each component, we convolve the ISDs with the source time function.

2.1.7. Master Equation for TFMCF

For a nonisotropic Gaussian ACF (2), the master equation (3) is written as

\[
o\tau_0 \partial_\tau \Gamma_2 + i \frac{k_z}{2k} \left( \frac{\partial^2}{\partial x_d^2} + \frac{\partial^2}{\partial y_d^2} \right) \partial \Gamma_2 + k_z^2 \pi^2 \alpha_z \left( \frac{\chi^2}{a_x^2} + \frac{\chi^2}{a_y^2} \right) \partial \Gamma_2 = 0.
\]

The r coordinate and the transverse coordinates \((x_d, y_d)\) are scaled by the receiver distance \( r \) with

\[
a_x = \sqrt{\pi \kappa_z^2 k_z r_0} = a_x \sqrt{\kappa_z^2 k_z} r_0
\]

and \((x_d, y_d) = (r/a_r)/(r_0) \) (\( \xi_{ad}, \xi_{yd} \)) are \( \xi_{ad}, \xi_{yd} \) are nondimensional coordinates on the transverse plane and \( \tau \) varies from 0 to 1. Then, equation (9) becomes

\[
o\tau_0 \partial_\tau \Gamma_2 + i \frac{\omega_j M_0}{\tau^2} \left( \frac{\partial^2}{\partial \xi_{ad}^2} + \frac{\partial^2}{\partial \xi_{yd}^2} \right) \partial \Gamma_2 + \left( \frac{\chi^2}{a_x^2} + \frac{\chi^2}{a_y^2} \right) \partial \Gamma_2 = 0.
\]

where

\[
t_{M0} = \sqrt{\pi \kappa_z^2 r_0}/(2V_0 a_x).
\]

The characteristic time for the x component at a distance \( r \) is written as

\[
t_{Mx} \equiv t_{M0} \left( \frac{a_x^2}{a_r^2} \right)^2 = \frac{\sqrt{\pi \kappa_z^2 a_x^2 r^2}}{2V_0 a_x^2} = \frac{\sqrt{\pi \kappa_z^2 a_x^2 k_z^2 r^2}}{2V_0 k_z} = \frac{1}{2k_z^2 a_x^2 V_0},
\]

where \( r/V_0 \) is the travel time at the distance \( r \). Taking the same procedure for the y component, we have

\[
t_{My} \equiv t_{M0} \left( \frac{a_y^2}{a_r^2} \right)^2 = \frac{\sqrt{\pi \kappa_z^2 a_y^2 r^2}}{2V_0 a_y^2} = \frac{\sqrt{\pi \kappa_z^2 a_y^2 k_z^2 r^2}}{2V_0 k_z} = \frac{1}{2k_z^2 a_y^2 V_0}.
\]

(15a)

(15b)

[21] Substitution of \( \partial \Gamma_2(x, r, y) = 0, y = 0 \) with the replacement \( r_0 \rightarrow r \) into (6) gives the reference ISD without the wandering effect at a distance \( r \):

\[
\hat{I}_0^R(r, t, \omega) = \frac{1}{2\pi \tau^2} \int_{-\infty}^{\infty} d\omega \omega_0 e^{-i\omega(t - r/V_0)} \frac{\partial_\omega \Gamma_2}{\sqrt{\sin \left( \frac{\partial_\omega \Gamma_2}{a_\omega} \right)}} \frac{\partial_\omega \Gamma_2}{\sqrt{\sin \left( \frac{\partial_\omega \Gamma_2}{a_\omega} \right)}}.
\]

(14)

(12)

(13)

[22] The product \( \frac{a_z}{a_r} \) can be written as

\[
\frac{a_z}{a_r} = \frac{2V_0 a_r k_z^2}{\tau^2} r
\]

The characteristic time for the x component at a distance \( r \) is written as

\[
t_{M0} \equiv t_{M0} \left( \frac{a_x^2}{a_r^2} \right)^2 = \frac{\sqrt{\pi \kappa_z^2 a_x^2 r^2}}{2V_0 a_x^2} = \frac{\sqrt{\pi \kappa_z^2 a_x^2 k_z^2 r^2}}{2V_0 k_z} = \frac{1}{2k_z^2 a_x^2 V_0},
\]

(15a)

(15b)

[23] In (15a) and (15b), we introduced coherence radii

\[
a_{ax} = a_x \sqrt{\sqrt{\pi \kappa_z^2 a_x^2 r^2}} \quad \text{and} \quad a_{ay} = a_y \sqrt{\sqrt{\pi \kappa_z^2 a_y^2 r^2}}.
\]

(16)

which satisfy \( k_z^2 r \) for large \( \kappa_z^2 a_r \). Coherence radii are used for scaling the distance in the transverse plane and for the study of the applicable limit.
2.1.8. Resultant ISDs

[24] Taking the second derivative of the TFMCF (12) with replacing \( r_0 \) with \( r \) and substituting it into (5a) and (5b) with \( \bar{w} = 1 \), we have the ISDs without the wandering effect for transverse components:

\[
\tilde{I}^p_{10}(r, t, \omega_c) = \left( \frac{V_0 M_0}{r} \right) \frac{1}{\pi r^2} \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega_j e^{-i\omega_j(t-r/V_0)} \left\{ \frac{\alpha_z}{\alpha_s} - \frac{\alpha_z}{\alpha_s} \right\} \sqrt{\frac{\alpha_z}{\alpha_s}} \left[ 1 - \left( \frac{\alpha_z}{\alpha_s} \right) \cot \left( \frac{\alpha_z}{\alpha_s} \right) \right].
\]

(17a)

and

\[
\tilde{I}^p_{10}(z, t, \omega_c) = \left( \frac{V_0 M_0}{r} \right) \frac{1}{\pi r^2} \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega_j e^{-i\omega_j(t-r/V_0)} \left\{ \frac{\alpha_z}{\alpha_s} - \frac{\alpha_z}{\alpha_s} \right\} \sqrt{\frac{\alpha_z}{\alpha_s}} \left[ 1 - \left( \frac{\alpha_z}{\alpha_s} \right) \cot \left( \frac{\alpha_z}{\alpha_s} \right) \right].
\]

(17b)

[25] The above integral representation can be numerically evaluated by using an FFT for any set of correlation distances. Substituting (17a) and (17b) into (5c) and using

\[
\frac{d \tilde{I}}{d \omega_d} = \frac{1}{\pi r^2} \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega_j e^{-i\omega_j(t-r/V_0)} \left\{ \frac{4V_0}{r} \left( -i \frac{d}{d \omega_d} \right) \right\}
\]

we have

\[
\tilde{I}^p_{20}(r, t, \omega_c) = \left( \frac{V_0 M_0}{r} \right) \frac{1}{\pi r^2} \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega_j e^{-i\omega_j(t-r/V_0)} \left\{ \frac{4V_0}{r} \left( -i \frac{d}{d \omega_d} \right) \right\}
\]

\[
= \left\{ \tilde{I}^R_{0}(r, t, \omega_c) - \frac{4V_0}{r} (t - r/V_0) \tilde{I}^R_{0}(r, t, \omega_c) \right\}
\]

\[
= \left\{ 1 - \frac{4V_0}{r} (t - r/V_0) \right\} \tilde{I}^R_{0}(r, t, \omega_c).
\]

(17c)

[26] We note that the resultant ISDs are practically independent of the central angular frequency.

2.1.9. Time Integral of ISD

[27] Using the inverse Fourier transform at \( \omega_d = 0 \), we have the time integral of the transverse component ISDs with geometrical spreading correction

\[
4\pi r^2 \int_{0}^{\infty} \tilde{I}^p_{10}(r, t, \omega_c) dt = \frac{2\sqrt{\pi\varepsilon^2}}{3} \frac{a_r}{\alpha_s}
\]

(18a)

and

\[
4\pi r^2 \int_{0}^{\infty} \tilde{I}^p_{10}(z, t, \omega_c) dt = \frac{2\sqrt{\pi\varepsilon^2}}{3} \frac{a_r}{\alpha_s}
\]

(18b)

By using \( 4\pi r^2 \int_{0}^{\infty} \tilde{I}^0_{0}(r, t, \omega_c) dt = 1 \), we have

\[
4\pi r^2 \int_{0}^{\infty} \tilde{I}^p_{0}(r, t, \omega_c) dt = 1 - \frac{2\sqrt{\pi\varepsilon^2}}{3} \frac{a_r}{\alpha_s} + \frac{a_r^2}{\alpha_s^2} + \frac{a_r^2}{\alpha_s^2}
\]

(18c)

[28] The time integral of the transverse component ISD with geometrical spreading correction linearly increases with increasing travel distance. This integral will be a good measure of randomness.

[29] Integrals (18a) and (18b) mean that the partition of energy to the transverse components, which should be small at a travel distances \( r \),

\[
e^2 \frac{a_r}{\alpha_s^2} \ll 1 \quad \text{and} \quad e^2 \frac{a_r}{\alpha_s^2} \ll 1
\]

(19)

because the parabolic approximation treats wave propagation within a small angle around the forward direction. These conditions restrict the application range of this approximation. These conditions are written as \( a_r/k_c > 1 \) and \( a_r/k_c > 1 \) by using (16), which are the same as that derived from the applicable condition of the Markov approximation for the mutual coherence function in stationary state [Rytov et al., 1989, p. 110].

2.1.10. Axially Symmetric ACF With Respect to the Global Ray Direction

[30] Fourier transforms (14) and (17a) and (17b) can be analytically evaluated for special cases of interest. When the randomness is isotropic (see Paper II), the reference ISD without wandering effect is written by using the elliptic theta function of the fourth kind as

\[
h(r, t - r/V_0, \tau_{00}) = \frac{1}{2\pi r^2} \int_{-\infty}^{+\infty} d\omega_j e^{-i\omega_j(t-r/V_0)} \frac{\sin \theta_d}{4\pi \sin \theta_d}
\]

where

\[
0\theta_d(q) = \theta_d(q)\left|_{u=0} = 8 \sum_{n=1}^{\infty} (-1)^{n+1} n^2 q^{2n}
\]

(MathWorld-A Wolfram Web Resource data are available at http://mathworld.wolfram.com/JacobiThetaFunctions.html). This solution for a scalar wavelet was first obtained by Shishov [1974]. The black trace in Figure 2 is 4\pi r^2 h plotted against reduced time \( t - r/V_0 \). It has a maximum peak of about 1.48/t_{00} at the reduced time 0.367t_{00}. The envelope time width is well characterized by \( \tau_{00} \), which is proportional to the square of travel distance and the reciprocal of the common correlation distance. In Figure 2, the reference ISD \( j \) for a plane wavelet (see Paper I) is plotted by a gray curve for comparison. The envelope width of a spherical wavelet is larger than that for a plane wavelet. The peak decay of a spherical wavelet with travel distance is faster than that of a plane wavelet because of the geometrical decay of the inverse square of travel distance.

[31] If the ACF is axially symmetric around the global ray direction along the z axis, writing the common correlation distance in the transverse plane as \( \alpha_s = a_r \), \( \alpha_s = a_r \), we can write the reference ISD as

\[
\tilde{I}^p_{0}(r, t, \omega_c) = \frac{1}{2\pi r^2} \int_{-\infty}^{+\infty} d\omega_j e^{-i\omega_j(t-r/V_0)} \frac{\alpha_s}{\alpha_s} \frac{\alpha_s}{\alpha_s} \frac{\alpha_s}{\alpha_s} \frac{\alpha_s}{\alpha_s} \frac{\alpha_s}{\alpha_s} \frac{\alpha_s}{\alpha_s}
\]

(20)

\[
= h(r, t - r/V_0, \tau_{00})
\]

(21)
by using the analytic solution $h$, where the characteristic time at a distance $r$ is given by

$$t_{M}^{\text{Axial}} = t_{M0} \left( \frac{a_z}{a_t} \right)^2 = \frac{\sqrt{\pi \varepsilon^2 a_r r^2}}{2V_0 a_r^2}$$

(22)

from (15a, b). The envelope broadening is proportional to the square of the aspect ratio $a_z/a_r$. In this case,

$$I_{10}^P(r, t, \omega_c) = I_{10}^P(r, t, \omega_c)$$

$$= \frac{2V_0}{r} \frac{1}{\pi r^2} \int_{-\infty}^{\infty} d\omega_\theta e^{-i\omega_\theta (t/r)} \left( \frac{a_z}{a_t} \right) \frac{1}{4\pi \sin \left( \frac{a_z}{a_t} \right)}$$

and

$$I_{20}^P(r, t, \omega_c) = \left[ 1 - \frac{4V_0}{r} (t - r/V_0) \right] h(r, t - r/V_0, t_{M}^{\text{Axial}}).$$

(23a)

and

Equation (22) is written as

$$t_{M}^{\text{Axial}}/(r/V_0) = \sqrt{\pi \varepsilon^2 a_r r^2/(2V_0^2 a_r^2)}.$$

The characteristic time means the time width of envelope. If the envelope is composed of small angle scattering waves, it should be smaller than the travel time, $\varepsilon^2 a_r r^2/(2V_0^2 a_r^2) < 1$. This condition is the same as equation (19) derived from the condition that the time integral of the transverse component power is small.

2.1.11. MS Envelopes of a $P$ Wavelet

[34] Figure 3a illustrates how each component of the MS vector-wave envelopes (ISDs including wandering effect) varies with travel distance increasing for impulsive $P$ wavelet radiation from a point source in random media ($V_0 = 6$ km/s) characterized by axially symmetric Gaussian ACFs with $\varepsilon = 5\%$ for (Figure 3a) $z$, (Figure 3b) $x$, and (Figure 3c) $y$ components. The plots of the transverse component (x and y components) traces are enlarged. Fine solid, broken and bold solid lines represent ISDs for different sets of correlation distances $\{a_z, a_r, a_r\} = \{5, 5, 5\}$ km (Ray V in Figure 1), $\{5, 5, 5\}$ km, and $\{2.5, 5, 5\}$ km (Ray H in Figure 1), respectively. Figure 4a shows a comparison of different component envelopes at a distance of 150 km for each set of correlation distances. All component traces show that the envelope width increases with travel distance increasing; however, the envelope width of the transverse component is larger than that of the $z$ component. The peak value for the transverse component decays more slowly than that for the $z$ component. In the case of isotropic randomness (broken lines in Figure 3), the peak decay of the $z$ component is approximated by $r^{-3}$ and that of the transverse component by $r^{-3}$. Since scattering effects dominate over the wandering effect at large travel distances (see Paper II). In the case that the correlation distance in the transverse directions are longer than that in the $z$ direction (fine solid lines in Figure 3), which corresponds to the Ray V in Figure 1, the envelopes in the $z$ component are sharp.
and those in the transverse components are small compared with broken lines for isotropic randomness. The x and y component traces are the same each other since the randomness is axially symmetric around the z axis (see the left bin in Figure 4a). The broadening effect in the left bin in Figure 4a is smaller than that in the center bin since the characteristic time is proportional to \((a_z/\alpha)^2\) (see (22)). When the correlation distance in the x direction is shorter than others (bold solid lines in Figure 3), which corresponds to the Ray H in Figure 1, the envelopes in the z component are broad and those in the x component are large compared with broken lines for isotropic randomness. The y component envelope is smaller than the x component envelope at each travel distance reflecting the inequality \(a_x < a_y\) (see the right bin in Figure 4a).

Figure 4b shows vector-wave envelopes for the case of three different correlation distances: \(\{a_x, a_y, a_z\} = \{5, 5, \text{2.5}\} \text{ km}, \{10, 2.5, 5\} \text{ km}, \text{and } \{2.5, 5, 10\} \text{ km}\). We may interpret different bins for different ray directions for the same nonisotropic random media since we choose the z direction as the global ray direction. All the three component envelopes are different each other. The envelope broadening of the z component is the shortest in the left bin and the largest in the right bin reflecting the length of the correlation distance in the z direction. The right bin shows that the excitation and the duration of the x component are especially large which reflects the shortest correlation distance in the x direction. The center bin shows that the excitation of the x component is the smallest since the correlation distance in the x direction is the largest. When the aspect ratio \(a_z/\alpha_x\) or \(a_z/\alpha_y\) is large, the accuracy of the Markov approximation becomes lower even if \(\varepsilon\) is small because the condition (19) is not fulfilled at large travel distances.

These simulations show that envelope broadening is especially sensitive to the random medium spectrum in the transverse plane. These envelopes simulated here agree with the characteristics of wavefields numerical simulated in nonisotropic random media by Hong and Wu [2005].
2.1.12. Relation to the Solution for 2-D Random Media

When we let the ratio \(a_z/a_y\) \(\rightarrow 0\),

\[
G_2(r, x_0) = 0 \quad \text{as} \quad a_z/a_y \rightarrow 0
\]

which is the solution for nonisotropic random media in 2-D as obtained by Saito [2006]. Furthermore, if we put the ratio \(a_z/a_x\) \(\rightarrow 1\),

\[
G_2(r, x_0) = 0 \quad \text{as} \quad a_z/a_x \rightarrow 1
\]

which is the solution for isotropic random media in 2-D as obtained by Saito [2006]. We note that the solution for isotropic random media in 2-D cannot be obtained as a limit of the solution for isotropic random media in 3-D.

2.2. S Wavelet

2.2.1. Intensity Spectral Densities

When the S wave velocity \(\beta(x) = V_0 (1 + \xi(x))\) has a small fractional fluctuation \(\xi(x)\) around the average velocity \(V_0\) and the wavelength is much smaller than any correlation distances of the random inhomogeneity, the displacement vector for the S wave is written as \(u = \nabla \times B\) by using the vector potential \(B\), which obeys

\[
\Delta B - \frac{1}{V_0^2} \partial_t^2 B + 2 \frac{1}{V_0^2} \xi(x) \partial_t B = 0. \tag{25}
\]

This equation means that the three components are independent. For radiation from a point source at the origin, outgoing S wave which is axially symmetric around the y axis and polarized to the x axis for a ray on the z axis can be well represented by a vector potential having the y component only \(B = (0, \phi, 0)\) even in inhomogeneous media, where \(u \equiv (u_x, u_y, u_z) = (-\partial_x \phi, 0, \partial_z \phi)\). Here we write the S wave potential as a superposition of spherical outgoing harmonic waves of angular frequency \(\omega\) as \(\phi = (1/2\pi) \int_{-\infty}^{\infty} (U(i/k_0)) e^{ikr - i\omega t} \, d\omega\), where \(k_0 = \omega/V_0\) is
the wave number. The field $U$ is governed by the parabolic equation (1) since the wavelength is shorter than the correlation distances.


\[ I^S_y(r, t, \omega_c) = 0, \]  

(26a)

and the $z$ component ISD is

\[ I^z_y(r, t, \omega_c) = \frac{1}{2 \pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\omega_x d\omega_y d\omega_z \text{e}^{-i\omega(t-x/V_0)} \tilde{W}(r, \omega_d) \left( \frac{1}{k_z^2} \right) \tilde{F}\left( \mathbf{x}_{t,d}, r, \omega_c, \omega_d \right) \bigg|_{x_d=0}. \]  

(26b)

By using the reference ISD $I^R$ defined by (14), we have the $x$ component ISD as

\[ I^x_y(r, t, \omega_c) = \frac{1}{2 \pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\omega_x d\omega_y d\omega_z \text{e}^{-i\omega(t-x/V_0)} \tilde{W}(r, \omega_d) \left( \frac{1}{k_z^2} \right) \tilde{F}\left( \mathbf{x}_{t,d}, r, \omega_c, \omega_d \right) \bigg|_{x_d=0}. \]  

(26c)

2.2.2. ISDs for Nonisotropic Gaussian ACF

The master equation for the TFMCF and the reference ISD are the same as those for $P$ waves. For radiation from a point source at the origin, we impose the initial condition (8), which means that the $y$ component of $S$ wave potential is isotropic around the origin. The corresponding ISDs are $I^S_{x0}(r, t, \omega_c) = I^S_{y0}(r, t, \omega_c) = 1/(4\pi r^2) \delta(t - r/V_0)$ and $I^S_{z0}(r, t, \omega_c) = 0$ at a small $r$ along the $z$ axis. Using the explicit representation of the TFMCF solution (12), we have the $S$ wave ISD of each component without the wandering effect

\[ I^S_{x0}(r, t, \omega_c) = \frac{V_0 \delta(tn_r)}{r} \frac{1}{2 \pi^2} \int \int \int d\omega_x d\omega_y d\omega_z \text{e}^{-i\omega(t-x/V_0)} \times \frac{\alpha_z}{\sqrt{\sin(\alpha_z/\omega_s)}} \sqrt{\frac{\alpha_z}{\alpha_z - \omega_s}} - \frac{1}{\alpha_z} \cos(\omega_s/\alpha_z) \right] \]  

(27a)

and

\[ I^S_{y0}(r, t, \omega_c) = \left[ 1 - \frac{4V_0}{r} \left( \frac{t - r}{V_0} \right) I^R_{y0}(r, t, \omega_c) \right]. \]  

(27b)

Replacing $V_0$ with the average $S$ wave velocity in (17a) and (17c), we have (27a) and (27b) for an $S$ wavelet. The characteristic times of the $S$ waves are larger than those of the $P$ waves modified by a factor of the average velocity ratio if the fractional fluctuation of the $P$ wave velocity is the same as that of the $S$ wave velocity.

3. Summary and Discussions

The broadening of the envelope and the excitation of the transverse (longitudinal) component for $P$ waves ($S$ waves) in earthquake seismograms are clear evidence of scattering due to random inhomogeneity in the Earth. Extending the Markov approximation for scalar waves, we developed the direct synthesis of vector-wave envelopes in 3-D random elastic media statistically characterized by a nonisotropic Gaussian ACF for the case that the wavelength is shorter than the correlation distances and the ray direction is parallel to one of the principal axes of the ACF. Spherically outgoing vector waves radiated from a point source in random elastic media could be a basic model of high-frequency seismogram envelopes from micro-earthquakes occurring in the inhomogeneous lithosphere. The stochastic master equation for the TFMCF of the potential field is analytically solved. The Fourier transform of the TFMCF gives intensity spectral densities, which are MS envelopes of band-pass filtered vector-wave traces. If the ACF is axially symmetric around the ray direction, MS envelopes of vector components are analytically solved. The aspect ratio of the correlation distance in the longitudinal direction to that in the transverse direction is the key parameter for the envelope broadening and the excitation in the orthogonal component. Envelope broadening becomes longer and the transverse (longitudinal) component amplitude increases faster for a $P$ wavelet (for an $S$ wavelet) when the correlation distance in the transverse plane becomes smaller. When the vertical correlation distance is shorter than the horizontal one, as seen in the real Earth, the envelope broadening is larger for horizontal raypaths compared with vertical raypaths.

The above formulation will be helpful for understanding seismic wave propagation through laminated random structures as used in models of the lower crust and the subducting oceanic slab. Interpretation of real data requires the development of more realistic theoretical models that can model random media characterized by a power law spectrum with a point shear dislocation source radiation. It will also be necessary to develop mathematics to treat the oblique incidence relative to the principal axes of the ACF. The applicable range of the Markov approximation should be quantitatively examined by a comparison with the numerical simulations in nonisotropic random elastic media.

Appendix A: Vector-Wave Envelopes in 3-D Nonisotropic Random Media for the Incidence of a Plane Wavelet

A.1. P Wavelet

We study the case of the incidence of an impulsive plane $P$ wavelet along the $z$ axis in a homogeneous medium with the $P$ wave velocity $V_P$ in a half-space of $z < 0$ onto a nonisotropic random medium with the $P$ wave velocity $V(x) = V_P(1 + \xi(x))$ in a half-space of $z > 0$. If the velocity fractional fluctuation $\xi(x)$ is small and the wavelength is shorter than any of the correlation distances, there is little conversion scattering. Then, the scalar potential of the $P$ wave at a distance $z$ in a random medium is written...
as a superposition of harmonic plane waves of angular frequency \( \omega, \phi = (1/2\pi) \int_{-\infty}^{\infty} (U(k_0)e^{i2\pi x'' + i2\pi y''}dk_0, \) where \( k_0 = \omega / V_0 \) is wave number. The field \( U \) is governed by a parabolic equation,

\[
2ik_0\partial_t U + \Delta_{\perp} U - 2k_0^2 \xi U = 0, \tag{A1}
\]

where \( \Delta_{\perp} \) is the Laplacian for the transverse coordinates \( x_{\perp} \). We imagine an ensemble of random media \( \{ \xi(x) \} \). By using the same procedure for the ensemble average as used by Paper I, we can define the ISD of each vector component as

\[
I^p_{\parallel}(z, t, \omega_z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \xi e^{-i\omega(t-z/V_0)} \tilde{w}(z, \omega) \cdot \left[ -\frac{1}{k^2} \partial^2_{\omega^2} \mathcal{G}(\mathbf{x}_{\perp}, z, \omega_z) \right]_{\omega=0}, \tag{A2a}
\]

\[
I^p_{\perp}(z, t, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \xi e^{-i\omega(t-z/V_0)} \tilde{w}(z, \omega) \cdot \left[ -\frac{1}{k^2} \partial^2_{\omega^2} \mathcal{G}(\mathbf{x}_{\perp}, z, \omega) \right]_{\omega=0}, \tag{A2b}
\]

and

\[
I^p_{\parallel}(z, t, \omega_z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \xi e^{-i\omega(t-z/V_0)} \tilde{w}(z, \omega) \cdot \left[ (1 + \Delta_{\perp}/k^2) \mathcal{G}(\mathbf{x}_{\perp}, r, \omega_z) \right]_{r=0} \cdot (I^p_{\parallel}(z, t, \omega_z) - I^p_{\perp}(z, t, \omega_z)). \tag{A2c}
\]

where \( \mathcal{G}(\mathbf{x}_{\perp}, r, \omega_z) \) is the TFMCF without the wandering effect and the function \( \tilde{w}(z, \omega_d) \equiv e^{-i\omega_0 k^2 \omega_d^2} \) is the wandering term at distance \( z \). We define the reference ISD as

\[
I^p_{\parallel}(z, t, \omega_z) \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \xi e^{-i\omega(t-z/V_0)} \tilde{w}(z, \omega) \cdot 0\mathcal{G}(\mathbf{x}_{\perp}, 0, \omega_d), \tag{A3}
\]

For a nonisotropic Gaussian ACF (4), we have the stochastic master equation according to the Markov approximation for \( \mathcal{G}(\mathbf{x}_{\perp}, r, \omega_z) \) as

\[
\partial_t \mathcal{G}_0 + i \frac{k_0}{2k^2} \left( \frac{\partial_x}{\partial y} + \frac{\partial_y}{\partial x} \right) \mathcal{G}_0 + k^2 \sqrt{(2\pi)^2 \sigma_z^2 \frac{\partial^2_{\omega^2}}{\partial z^2} + \frac{\partial^2_{\omega^2}}{\partial y^2}} \mathcal{G}_0 = 0. \tag{A4}
\]

We solve this equation for the initial condition

\[
\mathcal{G}_0(\mathbf{x}_{\perp}, z = 0, \omega_z, \omega_d) = 1, \tag{A5}
\]

where the corresponding initial condition for the ISD is \( I^p_0(z, t, \omega_z) \equiv I^p_{\parallel 0}(z, t, \omega_z) = 0 \) and \( I^p_{\parallel 0}(z, t, \omega_z) = I^p_{\parallel 0}(z, t, \omega_z) = 0 \). Then, we obtain

\[
\mathcal{G}(\mathbf{x}_{\perp}, z, \omega_z, \omega_d) = e^{-i\omega_0 \tan(\omega_d z)} \frac{\sigma_z^2 \tan(\omega_d z)}{\sigma_y^2 \tan(\omega_d z)} \frac{\cos(\omega_d z)}{\cos(\omega_z)} \frac{\cos(\omega_z)}{\cos(\omega_z)}, \tag{A6}
\]

with the characteristic time \( t_{\xi0} = \sqrt{\pi} \xi z^2 \xi_0^2 (2V_0 a_z) \) and \( s_0 = 2e^{n/4} \sqrt{t_{\xi0} \omega_0} \). The reference ISD without wandering effect is

\[
I^p_{\parallel 0}(z, t, \omega_z) \equiv \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \xi e^{-i\omega(t-z/V_0)} \frac{1}{\sqrt{\cos(\omega_z)}} \frac{1}{\cos(\omega_z)}. \tag{A7}
\]

Taking the second derivative of the TFMCF, we have the ISDs as follows:

\[
I^p_{\parallel 0}(z, t, \omega_z) = \frac{4V_0 a_z}{z} \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \xi e^{-i\omega(t-z/V_0)} \frac{a_z}{a_z \tan(\omega_z)} \frac{\cos(\omega_z)}{\cos(\omega_z)} \cos(\omega_z), \tag{A8a}
\]

and

\[
I^p_{\parallel 0}(z, t, \omega_z) = \frac{4V_0 a_z}{z} \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \xi e^{-i\omega(t-z/V_0)} \frac{a_z}{a_z \tan(\omega_z)} \frac{\cos(\omega_z)}{\cos(\omega_z)} \cos(\omega_z). \tag{A8b}
\]

The above integrals can be numerically integrated for any set of correlation distances by using an FFT. Using \( d_{\xi_0} = \frac{2p_{\xi_0}}{s_0} \), we have

\[
I^p_{\parallel 0}(z, t, \omega_z) = \left[ 1 - (t - z/V_0)^2 \right] \frac{4V_0}{z} I^p_{\parallel 0}(z, t, \omega_z). \tag{A8c}
\]

The resultant ISDs are practically independent of central angular frequency.

When the ACF is isotropic, the reference ISD is analytically represented (Paper I) as

\[
I(z, t - r/V_0, \omega_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \xi e^{-i\omega(z - r/V_0)} \cos(\omega_0) \left[ \frac{\pi}{4V_0 \omega_0} \right] \frac{\cos(\omega_0)}{\cos(\omega_0)} H \left( t - \frac{z}{V_0} \right). \tag{A9}
\]

where \( \theta_i(v, q) \equiv \frac{\partial}{\partial t} \theta_i(v, q) \) is the derivative of the elliptic theta function of the fist kind \( \theta_i \) with respect to \( v \), where \( \theta_i(v, q) \equiv 2 \sum_{n=0}^{\infty} (-1)^n q^{(n+1)/2} \sin((2n + 1)v) \) (see MathWorld-A Wolfram Web Resource data, available at http://mathworld.wolfram.com/JacobiThetaFunctions.html). The solution for isotropic Gaussian ACF was obtained by Sreenivasia et al. [1976]. A gray trace in Figure 2 shows
the plot of $f$ against reduced time $t - z/V_o$. It has a maximum peak of about 0.46/$t_{M0}$ at reduced time 0.67$t_{M0}$.

[54] If the ACF is axially symmetric around the $z$ axis, as $a_x = a_y = a_r$, we can obtain an analytic solution by using $t_{M0} = \frac{a_z^2}{a_r^2} = \frac{a_z^2}{a_r^2} \sqrt{\frac{\pi}{\xi^2} - z^2}$ at a distance $z$ in (A9) as

$$I_0^S(t, \omega_c) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{1}{\omega_c} \cos \left( \frac{a_z}{a_r} s_0 \right) ds_0 \exp \left( -\frac{a_z^2}{a_r^2} s_0^2 \right).$$

(A10)

The corresponding ISDs are

$$\tilde{I}_{\omega_0}^P(z, t, \omega) = \frac{2V_o}{z} (t - z/V_o) \cdot f(z, t - z/V_o, t_{M0}^{S\text{axial}}).$$

(A11a)

and

$$\tilde{I}_{\omega_0}^P(z, t, \omega) = \left( 1 - \left( t - z/V_o \right) \left( \frac{4V_o}{z} \right) \right) f(z, t - z/V_o, t_{M0}^{S\text{axial}}).$$

(A11b)

A2. $S$ Wavelet

[56] For the incidence of an impulsive plane $S$ wavelet polarized to the $x$ direction propagating to the $z$ direction from a homogenous medium for $z < 0$ onto a random medium in a half-space of $z > 0$, we can represent the $S$ wave by using a vector potential having the $y$ component $\phi$ only. When we write the potential as a superposition of plane waves in the random medium, the same as for a $P$ wavelet, the field $U$ obeys the parabolic wave equation (A1), where $V_0$ is the average $S$ wave velocity and $\xi$ is the fractional fluctuation of the $S$ wave velocity. Then, the $y$ component ISD is always zero,

$$I_y^S(z, t, \omega) = 0.$$  

(A12)

[57] Following the same procedure as for the $P$ wavelet case, we solve the same stochastic master equation (A4) for the TFMCF under the same initial condition as (A5), which corresponds to ISDs as $I_{\omega_0}^S(z, t, \omega) = \delta(t - z/V_0)$ and $I_{\omega_0}^S(z, t, \omega) = I_{\omega_0}^S(z, t, \omega) = 0$ at $z = 0$. For the case of nonisotropic Gaussian ACF, the master equation for the TFMCF and the reference ISD are the same as those for $P$ waves. Using the explicit representation of the reference ISD, we have $S$ wave ISDs without wandering effect as

$$\tilde{I}_{\omega_0}^S(z, t, \omega) = \left( \frac{4V_o}{z} \right) \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{1}{\omega_c} \cos \left( \frac{a_z}{a_r} s_0 \right) ds_0 \exp \left( -\frac{a_z^2}{a_r^2} s_0^2 \right),$$

(A13a)

and

$$\tilde{I}_{\omega_0}^S(z, t, \omega) = \left[ 1 - \left( \frac{t - z}{V_o} \right) \left( \frac{4V_o}{z} \right) \right] f(z, t - z/V_o, t_{M0}^{S\text{axial}}).$$

(A13b)

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H. Sato, Geophysics, Tohoku University, Aramaki-Aza-Aoba 6-3, Aoba-ku, Sendai-shi, Miyagi-ken 980-8578, Japan. (sato@zisin.geophys.tohoku.ac.jp)