Tomographic inversion of the peak delay times to reveal random velocity fluctuations in the lithosphere: method and application to northeastern Japan

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SUMMARY
This study proposes a new approach to estimate a 3-D distribution of power spectral density function (PSDF) of random velocity inhomogeneities in the lithosphere and applies this method to northeastern Japan. Our approach analyses the peak delay times of high-frequency S wave for microearthquakes at plural frequency bands on the basis of the Markov approximation for the parabolic wave equation, where the peak delay time is measured as the time lag from the S-wave onset to the maximal amplitude arrival. This peak delay time is appropriate to quantify the accumulated scattering effects due to random inhomogeneities without significant effect of the intrinsic absorption. The target region is divided into many blocks, each of which is characterized by a von Kármán-type PSDF with two parameters. One of the two parameters characterizes the spectral decay and another one constrains the absolute values of the PSDF at wavelengths shorter than the correlation distance. The peak delay times are calculated with these parameters along the unperturbed ray path by means of a recursive formula that is based on the Markov approximation for the parabolic wave equation. According to this formula, the spatial variation of spectral decay can be identified only by the frequency dependence of the peak delay times. Considering this characteristic, we propose a two-step approach to achieve a stable estimation of both parameters. The first step constructs an initial model with an explicit constraint for the spatial variation of frequency dependence of the peak delay times. The second step improves the first-step result by minimizing the residual of the peak delay times at all frequency bands. In the synthetic test, this two-step approach successfully improves the estimation of both parameters. We apply this method for seismograms (2–4 Hz, 4–8 Hz, 8–16 Hz and 16–32 Hz) observed in northeastern Japan and reveal that strongly inhomogeneous regions are related to the Quaternary volcano distribution and seismicity. We investigate the PSDFs of random inhomogeneities \( P(m) \) for the case in which the correlation distance is 5 km, where \( m \) is the wavenumber. At the depth of 40–60 km beneath most of the Quaternary volcanoes, \( P(m) \) are estimated as \( 0.017m^{-3.5} \) km\(^3\) \( \sim 0.035m^{-4.0} \) km\(^3\) at \( 0.5 < m < 50 \) km\(^{-1}\). A remarkable characteristic of these PSDFs is the weak spectral decay in the study area. This characteristic means that lateral variation of the PSDF becomes significant at large wavenumbers. On the other hand, a high seismicity area in the western Hidaka region in Hokkaido is characterized by a steep spectral gradient as \( P(m) = 0.023m^{-4.2} \sim 0.050m^{-4.2} \) km\(^3\) at \( 0.5 < m < 50 \) km\(^{-1}\). The PSDF in this region is larger than in its neighbours, regardless of the wavenumbers. From the comparison with other geophysical observations, we speculate that these strong inhomogeneities are generated by liquid inclusions and/or fractured structures. These regions commonly indicate stronger inhomogeneities than those in their surrounding regions for large wavenumbers \( (m > 15 \) km\(^{-1}\)\), and our approach detects that they are described by different combinations of the two parameters. It implies that the...
1 INTRODUCTION

Quantitative estimation of random velocity inhomogeneities in the lithosphere is an important subject in seismology that has been addressed by various stochastic approaches analysing seismic wave envelopes at frequencies higher than 1 Hz (e.g. Sato 1989; Scherbaum & Sato 1991; Hoshiba 1993; Gusev & Abubakirov 1999b). In the analyses of high-frequency seismic envelopes, we need to pay much attention to an appropriate separation of scattering due to random inhomogeneities and the energy loss due to intrinsic absorption. Recent developments of theoretical studies on wave propagation in random media (Sato & Fehler 1998 for detailed review) proposed some practical approaches to achieve such a separation. One of these approaches is an analysis of duration times and/or maximal amplitudes of direct wave envelopes of microearthquakes (e.g. Scherbaum & Sato 1991; Saito et al. 2005) and another one is the analysis of the spatiotemporal distribution of coda wave energy (e.g. Fehler et al. 1992; Hoshiba 1993). Both approaches have been successfully applied for the observed seismograms. However, most of these practical approaches have to assume spatially uniform distributions for both random velocity inhomogeneities and intrinsic absorption (e.g. Hoshiba 1993; Saito et al. 2005). The spatial variation of random velocity inhomogeneities or intrinsic absorption has been estimated only by assuming a uniform distribution for either of the two properties (e.g. Nishigami 1991; Asano & Hasegawa 2004; Yoshimoto et al. 2006).

Analyses of direct wave duration times and/or maximal amplitudes must be useful to estimate the medium inhomogeneities and absorption only in the vicinity of the unperturbed ray path. Various definitions of duration time have been proposed to achieve a stable measurement, because the direct waves are usually complex and incoherent wave trains (e.g. Sato 1989; Petukhin & Gusev 2003). Among various definitions of duration time, a peak delay time \( t_p \), which is defined as the time lag from the maximum amplitude arrival of \( S \)-wave envelope, is insensitive to the intrinsic absorption (e.g. Saito et al. 2002). This characteristic implies that analyses of the peak delay times can reveal the random inhomogeneities in the lithosphere without significant effect of intrinsic absorption. As a pioneer work in seismology, Gusev & Abubakirov (1999a,b) conducted an inversion analysis of the peak delay times to estimate a vertical profile of random inhomogeneities and concluded that inhomogeneity in the crust is stronger than in the uppermost mantle. Their linear inversion can be extended to estimate lateral variations of random inhomogeneities. However, their approach has to assume a Gaussian-type function for the power spectral density function (PSDF) of random velocity inhomogeneities. This assumption is not always appropriate to characterize the lithospheric random inhomogeneities. For example, well-log data show a power-law decay of the PSDF of velocity fluctuation (e.g. Wu et al. 1994; Shiomi et al. 1997). Array analysis of the amplitude and phase fluctuations of teleseismic \( P \) waves also concluded that a power-law-type PSDF is more appropriate to characterize the lithospheric inhomogeneities than a Gaussian PSDF (e.g. Flatté & Wu 1988). The limitation for the available PSDF in the approach of Gusev & Abubakirov (1999a,b) makes it impossible to take account of the frequency dependence of the peak delay times. In observed seismograms, frequency dependences of the peak delay time have been observed in some previous studies (e.g. Obara & Sato 1995; Takahashi et al. 2007). For example, the peak delay times of the waves propagating beneath the Quaternary volcanoes shows larger peak delay times at higher frequencies (Takahashi et al. 2007).

In our previous study (Takahashi et al. 2007), we phenomenologically evaluated the spatial distribution of random velocity inhomogeneities without assuming any functional forms for the PSDF of random inhomogeneities. This approach simply allots the observed peak delay times to blocks in space by following a simple rule that a small peak delay time means the absence of the strong inhomogeneities on its ray path. Our previous study detected strong inhomogeneities beneath the Quaternary volcanoes and in the high-seismicity region in northeastern Japan. Even though this method cannot convert the results into any physical parameters characterizing random inhomogeneities. New mathematical bases to evaluate the peak delay times from random inhomogeneities are required to overcome these problems.

A von Kármán-type PSDF having a power-law decay in short wavelengths can be a possible origin of the frequency dependence of envelope broadening, according to the studies of the Markov approximation for the parabolic wave equation (e.g. Saito et al. 2002). The Markov approximation for the parabolic wave equation stochastically describes multiple forward scattering in weakly inhomogeneous media (e.g. Ishimaru 1978; Rytov et al. 1989). In seismology, studies based on the Markov approximation have been developed for plane and spherical waves radiated from an impulsive source, assuming a Gaussian PSDF or a von Kármán-type PSDF for random inhomogeneities (Sato 1989; Saito et al. 2002). Scalar wave propagation assuming weakly anisotropic inhomogeneity is examined in 2-D random media (Saito 2006), and vector wave propagation in random media has been developed for a unified interpretation of three-component wave envelopes (e.g. Sato 2006). These theoretical studies have been successfully applied to interpret observed envelopes of seismic waves at regional distances (e.g. Obara & Sato 1995) and teleseismic distances (e.g. Kubanza et al. 2007).

In a recent work, Takahashi et al. (2008) examined the envelope broadening by assuming along-ray path variation of random inhomogeneities characterized by the von Kármán-type PSDFs and proposed a recursive formula to evaluate the peak delay time at a given travel distance from the statistical parameters of random inhomogeneities on the ray path. This recursive formula appropriately describes the travel distance and frequency dependences of the peak delay time in relation to the statistical parameters of the von Kármán-type PSDF. We may say that analyses of the peak delay time based on the recursive formula can reconstruct a more realistic model of the spatial variation of random velocity inhomogeneities.

On the basis of these backgrounds, this study proposes a new tomographic inversion of the peak delay times at plural frequency bands (2–4 Hz, 4–8 Hz, 8–16 Hz and 16–32 Hz) to estimate the 3-D distribution of the statistical parameters of random velocity inhomogeneities by assuming the von Kármán-type PSDFs. Since the peak delay time is quite insensitive to the intrinsic absorption,
we can expect this approach to reveal the spatial distribution of random inhomogeneities without any assumptions for the spatial distribution of intrinsic absorption. By conducting synthetic tests, we show that a two-step approach is appropriate to reconstruct the power-law characteristic of random inhomogeneities. Then, we apply our method to the observed seismograms in northeastern Japan and clarify the spatial distribution of random inhomogeneities in relation to the other geophysical observations. Finally, we discuss the possible origins of strong random inhomogeneities and possible developments of our study.

2 Forward Modelling: Peak Delay Time of Spherical Wave in Random Media Characterized by the Von Kármán-Type PSDFs

The S-wave velocity of an inhomogeneous medium is written as

\[ V(\mathbf{x}) = V_0 (1 + \xi(\mathbf{x})) , \]

where \( V_0 \) is the background velocity, and \( \xi(\mathbf{x}) \) is a small fractional fluctuation satisfying \( |\xi(\mathbf{x})| \ll 1 \). We consider an ensemble of random media. \( \xi(\mathbf{x}) \) is a random function of space and satisfies the condition that its ensemble average \( \langle \xi(\mathbf{x}) \rangle \) is zero. This study assumes a 3-D isotropic von Kármán-type PSDF to characterize the fractional fluctuation,

\[ P(m) = \frac{8\pi^{3/2}a^2\Gamma(k+3/2)}{\Gamma(k)(1+a^2m^2)^k}m^{k-3/2}, \tag{1} \]

where \( \Gamma \) is the gamma function, \( m \) is the wavenumber, \( \varepsilon \) is the root mean square (rms) of the fractional fluctuation, \( a \) is the correlation distance and \( \kappa \) is the parameter that controls the spectral decay in large wavenumbers \( (am \gg 1) \), that is, in short wavelengths (see Fig. 1). If the random inhomogeneities are described with a specific combination of \( \kappa, \varepsilon \) and \( a \) (i.e. uniform random medium), the peak delay time of the spherical waves impulsively radiated from a point source can be written as

\[ t_p = N_p(\kappa, \varepsilon, a) \times f^{2M_p(\kappa)+4} \times r_M(\kappa), \tag{2} \]

where \( r \) is the travel distance, and \( f \) is the predominant frequency. This representation is derived by the Markov approximation for the parabolic wave equation in weakly inhomogeneous random media (Saito et al. 2002). The parameters \( M_p(\kappa) \) and \( N_p(\kappa, \varepsilon, a) \) are related to the parameters characterizing the von Kármán-type PSDF as

\[ M_p(\kappa) = 1 + 2/p(\kappa), \tag{3} \]

and

\[ N_p(\kappa, \varepsilon, a) = b_p(\kappa) \times \left( \frac{C(k)\zeta^{2/(p-1)}(\varepsilon)}{2} \right) ^{1/2} \left( \frac{2}{\pi^{2/3}} \right) ^{2/3} \left( \pi^{2/3} \right) ^{1/2}, \tag{4} \]

The details of parameters \( b_p(\kappa), C(k) \) and \( p(\kappa) \) are summarized in Takahashi et al. (2008). Equations (3) and (4) suggest that \( \kappa \) and \( \varepsilon^{2(1/(p-1))}\varepsilon^{-1} \) (or \( M_p(\kappa) \) and \( N_p(\kappa, \varepsilon, a) \)) are control parameters that relate the peak delay time to the statistical characteristics of random inhomogeneities. We note that approaches based on the Markov approximation cannot separately estimate \( \varepsilon \) and \( a \). However, a combination of \( \varepsilon^{2(1/(p-1))}\varepsilon^{-1} \) and \( \kappa \) determines the PSDF at wavelengths shorter than the correlation distance, that is, at large wavenumbers (e.g. Saito et al. 2002; Takahashi et al. 2008). We may say that \( \varepsilon^{2(1/(p-1))}\varepsilon^{-1} \) is a parameter that constrains the absolute values of the PSDF at wavelengths shorter than the correlation distance.

For the case in which the random inhomogeneities along the ray path can be modelled by piecewise-constant variation of the stochastic parameters, the peak delay time can be evaluated by the recursive formula that we have proposed in the previous paper (Takahashi et al. 2008). Here, we briefly outline this method. We assume that the medium is composed of many zones, as shown in Fig. 2, and that the random inhomogeneity in each zone is characterized by a set of two parameters: \( \kappa \) and \( \varepsilon^{2(1/(p-1))}\varepsilon^{-1} \) (or \( M_p(\kappa) \) and \( N_p(\kappa, \varepsilon, a) \)). In the following, we use the term ‘non-uniform random medium’ for a medium that shows a piecewise-constant variation of the statistical parameters of the PSDF along the ray path. If the random inhomogeneity around the hypocentre is characterized by \( M^{(1)} \) and \( N^{(1)} \) up to the distance \( r_1 \) from the source (zone 1), the peak delay time at distance \( r_1 \) can be expressed as

\[ t_p^{(1)} = N^{(1)} \times f^{(2M^{(1)}+4)} \times r_1^{M^{(1)}}, \tag{5} \]

using eq. (2), where the subscript of \( r \) and superscript of \( M_p, N_p \) and \( t_p \) represent the number of zones counted from the seismic source. Then, we change a parameter of the statistical parameters of random inhomogeneities at distance \( r_1 \). The random inhomogeneities in zone 2 from distance \( r_1 \) to \( r_2 \) is characterized by \( M^{(2)} \) and \( N^{(2)} \). To evaluate the peak delay time at distance \( r_2 \), we replace the random inhomogeneities in zone 1 with those in zone 2 by introducing an equivalent travel distance \( r_1' \), which can be expressed as

\[ r_1' = \frac{r_p^{(1)}}{N^{(2)}(r_2')} \times f^{2M^{(2)}+4} \times r_1^{M^{(2)}}, \tag{6} \]

if parameters \( N_p \) and/or \( M_p \) changes from zone \((n-1) \) to zone \( n \) for the case of \( n \geq 2 \), the peak delay time can be evaluated by repeating the similar replacement from the source to the receiver sequentially. This procedure can be written in the following form:

\[ t_p^{(n)} = N_p^{(n)} \times f^{2M_p^{(n)+4}} \times r_1' \times (r_n - r_{n-1})^{M_p^{(n)}}, \]

where

\[ r_1' = \frac{r_p^{(n-1)}}{N_p^{(n)} \times f^{2M_p^{(n)+4}}} \times \left( \frac{r_n - r_{n-1}}{r_p^{(n-1)}} \right)^{1/M_p^{(n)}}, \tag{7} \]
From these formulations, we can summarize some remarkable characteristics of the peak delay times in relation to the stochastic parameters of the von Kármán-type PSDF as follows (see Saito et al. 2002; Takahashi et al. 2008 for details). In uniform media, the powers of travel distance and frequency dependences are controlled by ϵ only, as shown in eq. (2). The $M_p$ is 2.0 for $κ = 1.0$ and increases to 2.67 as $κ$ decreases to 0.1 (Saito et al. 2002). It means that the envelopes propagating at a uniform random medium characterized by small $κ$ show stronger broadening at higher frequencies. The parameters $ε$ and $α$ contribute to absolute values of the peak delay times without any effects for the frequency dependence. In non-uniform media, the effects of the statistical parameters for the peak delay time are quite different from those in uniform media. The power of travel distance dependence is affected by all of $M_p(κ)$ and $N_p(κ, ε, α)$ that are distributed along the ray path. The power of frequency dependence depends on $M_p(κ)$ along the ray path. If $κ$ decreases along the ray path, the peak delay times at higher frequencies increase more significantly.

3 TWO-STEP INVERSION

3.1 Objective functions and procedure

On the basis of the recursive formula, we construct a framework of our inversion analysis of the peak delay times to estimate the spatial distribution of the statistical parameters of the von Kármán-type PSDF. We are concerned with an inverse problem that is similar to the regional traveltime tomography (e.g. Aki & Lee 1976). The target region is divided into a number of blocks, and each block is characterized by two unknown statistical parameters, $κ$ and $ε$, and $α$ that are distributed along the ray path. The power of frequency dependence depends on $M_p(κ)$ along the ray path. If $κ$ decreases along the ray path, the peak delay times at higher frequencies increase more significantly.

![Figure 2. Schematic diagram of the recursive formula: (a) envelopes at travel distances $r_1$, $r_2$ and $r_n$. The peak delay times of these envelopes are indicated by horizontal arrows. (b) Along-ray path variation of the statistical parameters of random inhomogeneities. The random inhomogeneities in adjacent zones are different from each other: (1) for the original distribution of random inhomogeneities, (2) for the first replacement and (n) for all the replacements (see text for details).](image)

The frequency dependence of observed peak delay times has been investigated by a regression analysis $\log(t_p [f \text{ Hz}]/t_p [f_{\text{ref} \text{ Hz}}]) = A_{\text{freq}} + B_{\text{freq}} \log f$ at each station for many ray paths, because the scatter of the peak delay times is usually large (e.g. Obara & Sato 1995; Takahashi et al. 2007). The normalization by the peak delay time at the reference frequency $f_{\text{ref}}$ is introduced by Obara & Sato (1995) to reduce the effect of the travel distance dependence of the peak delay times. The choice of the reference frequency $f_{\text{ref}}$ depends on the analysed frequency range and data quality. Obara & Sato (1995) uses 2 Hz as the reference to normalize the peak delay times at 1 Hz, 2 Hz, 4 Hz and 8 Hz. Takahashi et al. (2007) chose the reference as 4–8 Hz for the data in 2–4 Hz, 4–8 Hz, 8–16 Hz and 16–32 Hz. These two studies reported that the $B_{\text{freq}}$ values tend to be large in the backarc side of the Quaternary volcanoes (or the volcanic front) in northeastern Japan. The large value of $B_{\text{freq}}$ implies the existence of small $κ$ values or the decrease of $κ$ on the ray paths. According to the recursive formula, we cannot directly interpret $B_{\text{freq}}$ as the along-ray path distribution of $κ$, but we can expect that an explicit constraint on $B_{\text{freq}}$ works effectively to stabilize the parameter estimation.

Considering these theoretical backgrounds and previous data analyses, we separate our inversion process into two steps to include $B_{\text{freq}}$ values in the objective function. In the first step, we construct an initial model from the peak delay times in the reference frequency ($f_{\text{ref}}$ Hz) and the $B_{\text{freq}}$ values. The objective function $S$ in the first step is defined as

$$S^{(1st)} = \sum_{ray(f_{\text{ref}} Hz)} \left( \frac{t_p^{\text{obs}} - t_p^{\text{calc}}}{\sigma_p} \right)^2 + w_{\text{freq}} N_{ray(f_{\text{ref}} Hz)} F_B(B_{\text{freq}}, \sigma_{\text{freq}}) + N_{ray(f_{\text{ref}} Hz)} [w_L L_x + w_L L_z].$$

(8)
The first term is the squared residual of the peak delay times at \( f_{\text{ref}} \) Hz that is normalized by the standard deviation \( \sigma_{\text{p}} \) of the peak delay times. The second term is the constraint term for the frequency dependence. The function \( F(B_{\text{freq}}, \sigma_{\text{Bfreq}}) \) is the squared residual of \( B_{\text{freq}} \) that is normalized by its standard deviation \( \sigma_{\text{Bfreq}} \) for all stations. The coefficient \( w_{B_{\text{freq}}} \) is a weighting factor for \( F(B_{\text{freq}}, \sigma_{\text{Bfreq}}) \). For example, \( F(B_{\text{freq}}, \sigma_{\text{Bfreq}}) \) in the synthetic test is defined as

\[
F(B_{\text{freq}}, \sigma_{\text{Bfreq}}) = \frac{1}{N_{s}} \sum_{j=1}^{N_{s}} \left( \frac{p_{\text{obs}}(j) - B_{\text{calc}}(j)}{\sigma_{\text{Bfreq}}(j)} \right)^2,
\]

(9)

where \( j \) represents the index for stations, and \( N_{s} \) is the number of all stations. \( N_{\text{ray(refHz)}} \) represents the number of ray paths at the reference frequency band. The third term in eq. (8) is composed of the Laplacian constraints for two unknown parameters in the horizontal direction (e.g. Yoshida & Koketsu 1990) to obtain a smooth distribution of both parameters:

\[
L_{s} = \frac{1}{N_{\text{blocks}}} \sum_{\text{blocks}} (\nabla^{2} \kappa)^2,
\]

(10)

and

\[
L_{c} = \frac{1}{N_{\text{blocks}}} \sum_{\text{blocks}} \left( \nabla^{2} \log \left( e^{2/(p-1)a^{-1}} \right) \right)^2.
\]

(11)

where \( N_{\text{blocks}} \) represents the number of blocks. The coefficients \( w_{s} \) and \( w_{c} \) are the weighting factors for the smoothing constraints.

We should note that our inversion does not give a smoothing constraint in the vertical direction for both the synthetic test and the data analysis. The synthetic test assumes a 2.5-D model without vertical variation of random inhomogeneities. In the case of the analysis in northeastern Japan, the constraint in the vertical direction does not work effectively to improve inversion convergences. The reason may be that the thickness of blocks in the vertical direction is too large to be constrained. It will be discussed at the end of Section 5.1.

The second step minimizes the residual of the peak delay times at all frequency bands under the constraints for the spatial smoothness and the first-step result. The objective function \( S \) used in the second step is

\[
S_{\text{2nd}} = \sum_{\text{freq}} \sum_{\text{ray(freqHz)}} \left( \frac{p_{\text{obs}} - p_{\text{calc}}}{\sigma_{\text{p}}} \right)^2 + N_{\text{ray(refHz)}} \left[ w_{s}L_{s} + w_{c}L_{c} \right]
+
\left( w_{1\text{stStep}}N_{\text{ray(freqHz)}} \right) \left[ w_{c} \sum_{\text{blocks}} \left( \kappa_{1\text{stStep}} - \kappa \right)^2 \right]
+
\left( w_{1\text{stStep}}N_{\text{ray(freqHz)}} \right) \left[ w_{c} \sum_{\text{blocks}} \left( \kappa_{1\text{stStep}} - \kappa \right) \right]^2
+
\left[ w_{c} \sum_{\text{blocks}} \left( \log \left( e^{2/(p-1)a^{-1}} \right) \right)^2 \right].
\]

(12)

The first term is replaced with the residual of the peak delay times at all frequency bands. The second term, the Laplacian constraints, is the same as in the first step. Third term weighted by \( w_{1\text{stStep}} \) is composed of the constraints for the first-step result. Note that the constraint term for \( B_{\text{freq}} \) is excluded from the objective function.

The objective functions in our inversion contain many unknown parameters. In addition, the objective functions are nonlinear with respect to the unknown parameters, and no appropriate initial model is available, except at the forearc side (east side) of the volcanic front in northeastern Japan (Saito et al. 2005). Because of these difficult conditions, we apply the genetic algorithm (GA) (Holland 1975) to find the optimum solution. This algorithm is one of the random searches that can start inversion analyses with randomly generated initial models. The GA seeks an optimum solution in the model parameter space by iterating the reproduction, crossover and mutation. To achieve a stable and fast convergence of this random search, we incorporate the concept of the neighbourhood algorithm (NA) (Sambridge 1999) into the GA. The NA makes possible a dense sampling in the parameter space near the optimal models. We also modify crossover and mutation processes in the GA to reach an optimum solution effectively. The details of the implementation of the NA and modifications of the GA are described in the Appendix.

The parameters used in the GA are determined after some trial analyses for both the synthetic data and the observed seismograms as follows. The number of genes in the GA is 100. The total number of generation is fixed as 25 000. We used 20 000 generations for the first step and 5000 generations for the second step. We assume \( \sigma_{\text{p}} = 1.0 \). The crossover is conducted for the reproduced genes under the probability of 80 per cent. Model parameters are randomly generated in the following ranges: \( 0.1 \leq \kappa \leq 0.9 \) and \( -6.5 \leq \log(e^{2/(p-1)a^{-1}}) \leq -0.5 \). We note that we choose a relatively small number of genes to save the computational time (approximately 7 hr for data analysis in northeastern Japan using eight cores of Intel Xeon 3.0 GHz) because of slow convergence.

### 3.2 Synthetic test in 2.5-D model

We conduct synthetic tests to examine how appropriately we can reconstruct the two unknown parameters by the above-mentioned approach. We assume that sources and receivers are on the xy-plane in an infinite 3-D medium (Fig. 3). An anomaly of strong inhomogeneities is set at \( x = 80 \sim 120 \) km, \( y = 80 \sim 120 \) km. This anomaly is characterized by \( \kappa = 0.5 \) and \( e^{2/(p-1)a^{-1}} = 10^{-42} \) km\(^{-1}\). The other region is \( \kappa = 0.7 \) and \( e^{2/(p-1)a^{-1}} = 10^{-36} \) km\(^{-1}\). The correlation distance and background velocity in this model are \( a = 5 \) km and \( V_{0} = 4.0 \) km s\(^{-1}\), respectively. We assume an infinite extent of this 2-D structure in the z-axis direction, that is, a 2.5-D model. Figs 3(a) and (b) show the distributions of \( \kappa \) and \( e^{2/(p-1)a^{-1}} \) of the original model, respectively. Fig. 3(c) represents the PSDFs at the wavenumber \( m = 15 \) km\(^{-1}\) (approximately corresponds to 10-Hz \( S \) wave) that are calculated from the two parameters using eq. (1), where we assume \( a = 5 \) km. We can expect the PSDF at the wavenumber \( m = 15 \) km\(^{-1}\) (a vertical broken line in Fig. 1) to represent faithfully a scattering strength for frequencies higher than 10 Hz, since the diffraction and multiple forward scattering of seismic waves are mainly caused by the inhomogeneities in which the spatial scale is longer than the wavelength of the incident wave.

The peak delay time at each station is calculated from the recursive formula (Takahashi et al. 2008) for isotropic radiation of spherical wave from a point source. Point sources are located on \( y = 0 \) km, with an interval of 10 km, and receivers are set on \( y = 130 \) km and \( y = 160 \) km, with an interval of 20 km. Station separation distances are almost the same as in the seismic data analysis in this study. The numbers of sources and stations are 21 and 14, respectively. Four frequency bands are used in this synthetic test: 2–4 Hz, 4–8 Hz, 8–16 Hz and 16–32 Hz. We note that our method takes account of the random inhomogeneities only in the xy-plane. The peak delay times for all of the data are 0.69 s \( \sim \) 2.75 s. For example, the peak delay times at a station (\( x = 90 \) km, \( y = 160 \) km) behind the anomaly are 1. 08 s \( \sim \) 1. 57 s at 2–4 Hz and 1. 22 s \( \sim \) 1. 75 s at 16–32 Hz. At another station (\( x = 30 \) km, \( y = 160 \) km),

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Figure 3. Map views of the input model used for the synthetic test: (a) $\kappa$, (b) $\varepsilon^2/(\kappa-1)\tau^{-1}$ and (c) PSDF (at $m = 15\text{ km}^{-1}$). Black dots and squares are receivers and sources, respectively. (d) Lateral variation of $B_{\text{freq}}$ at each station measured from all of the peak delay times. Open circles and squares are $B_{\text{freq}}$ values at $y = 160\text{ km}$ and $y = 130\text{ km}$, respectively.

The peak delay times are $1.06\text{ s} \sim 2.33\text{ s}$ at 2–4 Hz and $1.18\text{ s} \sim 2.75\text{ s}$ at 16–32 Hz. Fig. 3(d) shows the spatial variation of $B_{\text{freq}}$. The reference frequency $f_{\text{ref}}$ is set at 4–8 Hz, which is the same as in the observed seismic data analysis. The $B_{\text{freq}}$ values at $y = 160\text{ km}$ show a more strong lateral variation than those at $y = 160\text{ km}$. This is because the frequency dependence of $t_p$ can be weakened as travel distance increases for the case in which $\kappa$ becomes large on the ray path (see fig. 10a in Takahashi et al. 2008). Another reason is that the stations at $y = 160\text{ km}$ behind the anomaly can receive the waves that do not propagate in the strongly inhomogeneous region. We note that the following synthetic test is conducted without adding random noise in the peak delay times, since the objective of this synthetic test is to examine whether or not our two-step inversion can reconstruct the input model and improve the estimation of $\kappa$.

We divide the target region into $8 \times 10$ blocks, each of which has a size of 20 km $\times$ 20 km. To obtain an optimum inversion result, we conduct our inversion for various combinations of weighting factors in eq. (8) and find appropriate ones by examining the convergence of inversion. First, we determine the appropriate ranges of $w_\kappa$, $w_\varepsilon$ and $w_{B_{\text{freq}}}$ from the convergence at the end of the first-step inversion. Figs 4(a)–(c) are the plots of $L_\kappa$, $L_\varepsilon$ and $F(B_{\text{freq}}, \sigma_{B_{\text{freq}}})$ at the end of the first step against their weighing factors, respectively. Since the convergences of these constraints can be affected by each other, for example, $F(B_{\text{freq}}, \sigma_{B_{\text{freq}}})$ can be small if $L_\kappa$ remains large, we use different symbols in each plot to distinguish the convergences of the other constraints. Black dots represent the cases in which the residuals for the other constraints are sufficiently converged into small values. Grey symbols are the opposite cases. We can notice that black dots in these plots converge into constant values as each of the three weighting factors increases. This suggests that we can determine the lower limits of the weighting factors: $w_\kappa \geq 0.15$, $w_\varepsilon \geq 0.05$ and $w_{B_{\text{freq}}} \geq 0.005$.

Then, we estimate the appropriate $w_{1\text{step}}$ to achieve a stable improvement of the distribution of two unknown parameters in the second step. Since $F(B_{\text{req}}, \sigma_{B_{\text{req}}})$ is not included in the objective function in the second step, the role of $w_{1\text{step}}$ is to keep $F(B_{\text{req}}, \sigma_{B_{\text{req}}})$ small and secure a stable convergence of the inversion. For example, if $w_{1\text{step}}$ is extremely large, $F(B_{\text{req}}, \sigma_{B_{\text{req}}})$ is nearly constant during the second step without any improvement of the inversion result. On the other hand, in the case of significantly small $w_{1\text{step}}$, $F(B_{\text{req}}, \sigma_{B_{\text{req}}})$ rapidly increases as the iteration count increases. In other words, the result becomes far different from the first-step result. By taking account of such a characteristic of our approach, we examine the inversion convergence over the broad ranges for all weighting factors of $w_\kappa$, $w_\varepsilon$, $w_{B_{\text{req}}}$ and $w_{1\text{step}}$: $w_\kappa \geq 0.15$, $w_\varepsilon \geq 0.05$, $w_{B_{\text{req}}} \geq 0.005$ and $0.0001 \leq w_{1\text{step}} \leq 10000.0$. Fig. 4(d) shows the plots of $\delta F$ and $\delta t_p$ for $w_\kappa = 0.16$, $w_\varepsilon = 0.06$, $w_{B_{\text{req}}} = 0.005$ and $0.1 \leq w_{1\text{step}} \leq 100.0$, where $\delta F$
is the difference in $F(B_{freq}, \sigma_{B_{freq}})$ from the first-step result, and $\delta_{F}$ is the difference in the rms residual of the peak delay times for all frequency bands from the first-step result. The second step inversion mainly fits the peak delay times for all frequency bands; hence, $\delta_{F}$ comes to decrease, whereas $\delta_{F}$ generally increases. Fig. 4(d) shows that $\delta_{F}$ do not change significantly for $w_{1stStep} \geq 100$, but start to decrease from around $w_{1stStep} = 10$. $\delta F$ also changes at around $w_{1stStep} = 10$: $\delta F$ is quite small for $w_{1stStep} \geq 100$ and increases with decreasing $w_{1stStep}$ from $w_{1stStep} = 10$. It is necessary to find the solution that can explain both the peak time delays at four frequency bands and their frequency dependence observed at each station; hence, we choose the weighing factor of $w_{1stStep} = 5$, where both $\delta_{F}$ and $\delta F$ sufficiently change from the results of the first-step inversion. Then, fixing $w_{1stStep} = 5$, we execute the second inversion for $w_{s} \geq 0.15$, $w_{s} \geq 0.05$ and $w_{B_{freq}} \geq 0.005$ that are previously determined in the first-step inversion. Finally, we choose a solution that takes the minimum value of $S^{(2nd)}$ at the end of the second step among all trials for the range of $w_{s} \geq 0.15$, $w_{s} \geq 0.05$, $w_{B_{freq}} \geq 0.005$ and $w_{1stStep} = 5$. The weighting factors of the selected inversion result are $w_{s} = 0.16$, $w_{s} = 0.06$, $w_{B_{freq}} = 0.005$ and $w_{1stStep} = 5.0$.

Inversion result for this synthetic test is shown in Fig. 5. The rms residual of the peak delay times for all frequency bands (first term in eq. (12)) decreases to 0.015 s. Figs 5(a)–(c) are the spatial distributions of estimated $\kappa$, $e^{2/(\hat{\kappa} \kappa^{(p-e)}-1)} \kappa^{-1}$, and the PSDF, respectively. We evaluate the PSDF from the estimated $\kappa$ and $e^{2/(\hat{\kappa} \kappa^{(p-e)}-1)} \kappa^{-1}$ by assuming $a = 5$ km. All of the quantities are appropriately reconstructed, except at the vicinity of the anomaly and margins of the study area. The overestimation of $\kappa$ at the anomaly is compensated by large values for $e^{2/(\hat{\kappa} \kappa^{(p-e)}-1)} \kappa^{-1}$. The frequency dependence of reconstructed model is similar to that of the given model.

Fig. 6 is the result of inversion analysis without any constraints for the frequency dependence of the peak delay time; in other word, the objective function defined by eq. (12) with $w_{1stStep} = 0$ is used throughout the whole inversion process. We use the same values of $w_{s}$ and $w_{s}$ with the previous synthetic test shown in Fig. 5, but the result is quite different with the original structure. We note that the residual of the peak delay times in this synthetic test is almost the same as in the previous one. However, the frequency dependence (Fig. 6d) is significantly different from the given model (Fig. 3d). These results clearly indicate that inversion without any explicit constraints for the frequency dependence cannot reconstruct the spatial distribution of $\kappa$ and cannot converge into a unique result because of a trade-off between the two parameters. This is because the difference in the peak delay times among different frequency bands is masked by the difference in the ray path lengths and spatial variation of random inhomogeneities. This characteristic could make it difficult to reconstruct the anomaly of $\kappa$ only by minimizing the residual of the peak delay times at all frequency bands.

To know the stability of the two-step approaches, we examine the effects of the three weighting factors. Even if the weighting factors $w_{s}$, $w_{s}$ and $w_{B_{freq}}$ change with a factor of 1.5, the results show similar distribution of both parameters as long as the objective function at the end of the second step is sufficiently small.

Here, we mention about the effects of our modifications on the GA. Analyses without the NA process requires larger weighting factors for the Laplacian constraints to achieve a similar smooth distribution of the parameters in the same iteration counts. If we can spend twice or longer time for an inversion, we may be able to obtain similar results. However, such an approach is not preferable to decide the weighting factors. The analysis without modification for mutation and crossover cannot converge into a similar smooth distribution. If we use much larger weights for the Laplacian constraints, the inversion result can be a smooth distribution.
Figure 5. Map views of the reconstructed model in the synthetic test with an explicit constraint for frequency dependence: (a) $\kappa$, (b) $e^{2/\rho(c-1)}a^{-1}$ and (c) PSDF (at $m = 15$ km$^{-1}$, for $a = 5$ km). (d) Lateral variation of $B_{\text{freq}}$ at each station measured from all of the peak delay times synthesized from the reconstructed model. Grey colour means the absence of ray paths. Other symbols are the same as in Fig. 3.

with significantly large residuals of the peak delay time and $F(B_{\text{freq}}, \sigma_{B_{\text{freq}}})$.

Consequently, these synthetic tests make it clear that our two-step inversion approach is one of the practical approaches to estimate the spatial distribution of random inhomogeneities characterized by the von Karman-type PSDFs. The constraint for $B_{\text{freq}}$ works appropriately to reduce a trade-off between two unknown parameters. Since the $B_{\text{freq}}$ values have been stably estimated from observations (Obara & Sato 1995; Takahashi et al. 2007), we can expect that this approach can work effectively for the seismic data analysis.

4 SEISMIC DATA IN NORTHEASTERN JAPAN

We apply our inversion method developed above for the seismograms observed in northeastern Japan. Earthquake epicentres, seismic stations and ray paths used in this study are shown in Fig. 7. The velocity seismograms are recorded at 100-Hz sampling rate by Hi-net (High Sensitivity Seismograph Network Japan) of the National Research Institute for Earth Science and Disaster Prevention (Obara et al. 2005). The data set is the same as that used in Takahashi et al. (2007). Waveform data of 393 small- and moderate-sized earthquakes ($M_{\text{2.3}}$–5.5) are recorded at 262 stations. These earthquakes occurred around the subducting Pacific plate. Hypocentre locations are referred from the unified hypocentre list provided by the Japan Meteorological Agency (JMA). Focal depths of analysed earthquakes are larger than 35 km. We eliminate shallow earthquakes to avoid head waves and guided waves. This is because the studies based on the Markov approximation have been developed by assuming uniform background velocity.

We briefly describe how to measure the peak delay time in this study: We compose the rms envelopes for the sum of two horizontal components of velocity seismograms at four frequency bands (2–4 Hz, 4–8 Hz, 8–16 Hz and 16–32 Hz) after the deconvolution of the recording system response. The envelopes are smoothed by applying a moving time window whose width is twice the centre period of each frequency band. Then, the peak delay time in seconds is measured in a 30-s time window starting from the $S$-wave onset (see fig. 2 in Takahashi et al. 2007). We use the data showing clear $S$-wave onset and peak arrivals for hypocentral distances from 100 km to 250 km. The total numbers of waveform data that we used are 10738 (2–4 Hz), 11007 (4–8 Hz), 10832 (8–16 Hz) and 10118 (16–32 Hz). The number of data tends to be smaller in the higher-frequency ranges. This is because unclear peak arrivals due to strong scattering (and/or strong absorption) are frequently observed at higher frequencies. These data are eliminated from the original data set. We should note that there is arbitrariness for the time window length of envelope smoothing. In this study, the peak delay times varies from 0.1 s to 30 s. We choose a relatively short time window to avoid strong smoothing for an impulsive $S$ wave.
The remarkable characteristics of observed peak delay times are as follows. Those observed in the forearc side usually show small peak delay times both at lower and at higher frequencies. For the case in which seismic waves propagate beneath the Quaternary volcanoes, the peak delay times tend to be larger at higher frequencies. The waves propagating in the high-seismicity region beneath the western Hidaka show large peak delay time at all frequency bands (see Takahashi et al. 2007 for details).

We investigate the power of frequency dependence from the regression analysis $\log(t_p/f_{Hz})/\log(f_{ref}/f_{Hz}) = A_{freq} + B_{freq} \log f$ at each station for the focal depth ranges (35–50 km, 50–80 km and 80–200 km). Considering the numbers of data at each frequency band, we choose the reference frequency $f_{ref}$ as 4–8 Hz. This regression analysis at each station is conducted for the case in which the number of $t_p$ for all frequency bands is 16 or larger. Fig. 8 shows the spatial distributions of $B_{freq}$. The values of $B_{freq}$ for shallow events (35–50 km) are small at most of the stations, except for the backarc side of the Quaternary volcanoes. The $B_{freq}$ values for deep events tend to be large in the backarc side of the volcanic front. The depth dependence of $B_{freq}$ implies that the $\kappa$ value is large at the shallower part, except at the vicinity of the Quaternary volcanoes. These characteristics imply small values of $\kappa$ beneath the Quaternary volcanoes and/or the backarc side in deeper parts. We define the function $F(B_{freq}, \sigma_{B_{freq}})$ in the first-step inversion (eq. (8)) for the analysis in northeastern Japan instead of eq. (9) as

$$F(B_{freq}, \sigma_{B_{freq}}) = \frac{1}{\sum_{j=1}^{N_j(D)} \frac{1}{\sigma_{B_{freq}}(D, j)^2}} \sum_{D=1}^{3} \sum_{j=1}^{N_j(D)} \frac{(B_{obs}(D, j) - B_{calc}(D, j))^2}{\sigma_{B_{freq}}(D, j)^2},$$

(13)

where $j$ and $D$ are indices of stations and focal depth ranges ($D = 1, 2, 3$), respectively, and $N_j(D)$ is the number of observed $B_{freq}$ for three different depth ranges.

The study area is divided into 2617 blocks whose sizes are $0.30^\circ \times 0.30^\circ$ in the horizontal direction and 20 km in depth at the 0–20 km depth, and $0.20^\circ \times 0.20^\circ$ in the horizontal direction and 20 km in depth in deeper parts. The ray paths are calculated by the 1-D velocity model used for the routine hypocentre determination in Tohoku University (Hasegawa et al. 1978). We assume $V_0 = 4.0 \text{ km}^{-1}$ to calculate the peak delay time from $\varepsilon^{2/(\rho\kappa^2-1)\mu^{-1}}$ and $\kappa$.

5 INVERSION RESULT IN NORTHEASTERN JAPAN

5.1 Overview of inversion result

We show the inversion result in northeastern Japan. As we have done in the synthetic test, we first estimate the ranges of $w_\kappa$, $w_\varepsilon$ and $w_{B_{freq}}$ from the first-step results. Figs 9(a)–(c) are the plots

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Figure 6. Map views of the reconstructed model in the synthetic test without any constraints for the frequency dependence of the peak delay times: (a) $\kappa$, (b) $\varepsilon^{2/(\rho\kappa^2-1)\mu^{-1}}$ and (c) PSDF. (d) Lateral variation of $B_{freq}$ at each station measured from all of the peak delay times synthesized from the reconstructed model. Grey colour means the absence of ray paths. Other symbols are the same as in Fig. 3.
The Laplacian constraints for $\kappa$ and $g^{2/(n(x)-1)}a^{-1}$ in Figs 9(a) and (b) are nearly constant for $w_\kappa \geq 200$ and $w_\kappa \geq 3.8$, respectively. $F(B_{\text{freq}}, \sigma_{B_{\text{freq}}})$ in Fig 9(c) are almost constant for $w_{B_{\text{freq}}} \geq 3.0$. Then, we examine the inversion convergence over the broad ranges for all weighting factors of $w_\kappa$, $w_\varepsilon$, $w_{B_{\text{freq}}}$ and $w_{1\text{stStep}}$: $200.0 \leq w_\kappa \leq 600.0$, $3.8 \leq w_\varepsilon \leq 7.0$, $3.0 \leq w_{B_{\text{freq}}} \leq 8.0$ and $0.0001 \leq w_{1\text{stStep}} \leq 10000.0$. Fig. 9(d) shows the plots of $\delta F$ and $\delta t_p$ are the same as in the synthetic tests. Fig. 9(d) shows that $\delta t_p$ does not change significantly for $w_{1\text{stStep}} \geq 100$, but starts to decrease from around $w_{1\text{stStep}} = 10$. $\delta F$ is quite small for $w_{1\text{stStep}} \geq 100$ and increases with decreasing $w_{1\text{stStep}}$ from $w_{1\text{stStep}} = 1.0$. Following the procedure in the synthetic tests, we decide the weighing factor of $w_{1\text{stStep}} = 0.2$. Then, fixing $w_{1\text{stStep}} = 0.2$, we execute the second-step inversion for $w_\kappa \geq 200$, $w_\varepsilon \geq 3.8$ and $w_{B_{\text{freq}}} \geq 3.0$ that we determined in the first-step inversion. Finally, we choose a solution that takes the minimum value of $S(2\text{nd})$ at the end of the second step. The selected solution is the result that is estimated with $w_\kappa = 210.0$, $w_\varepsilon = 3.9$, $w_{B_{\text{freq}}} = 3.0$ and $w_{1\text{stStep}} = 0.2$.

Fig. 10 shows the rms residual of the peak delay times (first term in eq. (12)) for all frequency bands against the iteration count. The residuals from 100–25000 iterations are shown in this plot, since the residuals in the first generations are too large ($\sim 1000$ s). We can confirm the stable convergence in both steps. The rms residual of the peak delay times at all frequency bands converges to 2.60 s. The rms residuals at four frequency bands are 3.12 s for 2–4 Hz, 2.21 s for 4–8 Hz, 2.23 s for 8–16 Hz and 2.35 s for 16–32 Hz. The significantly large residual in 2–4 Hz might be due to the large data scatter, as shown in Takahashi et al. (2007). We note that the peak delay time measured from individual seismogram has inherently large scatter because this quantity is measured from incoherent wave train. This large scatter of the peak delay time is also the origin of slow convergence of the inversion.

Fig. 11 shows the map views of the best-fit $\kappa$, $g^{2/(n(x)-1)}a^{-1}$ and the PSDF at the wavenumber $m = 15 \text{ km}^{-1}$ for different depth ranges: (a) 0–20 km, (b) 20–40 km and (c) 40–60 km. We assume $a = 5 \text{ km}$

Figure 7. Distribution of seismic stations of Hi-net (dots) and epicentres (circles) used in this study in northeastern Japan. The diameter and grey scale of each circle represent the earthquake magnitude and focal depth, respectively. Grey lines are seismic ray paths. Open triangles represent Quaternary volcanoes.

Figure 8. Map view of the frequency dependence of the peak delay time in northeastern Japan for three focal depth ranges: (a) 35–50 km, (b) 50–80 km and (c) 80–200 km. The size of each circle or square represents the regression coefficient $B_{\text{freq}}$. Grey triangles are Quaternary volcanoes.
to evaluate the PSDF at $m = 15 \text{ km}^{-1}$. Fig. 12 shows the vertical cross-sections of $\kappa$, $e^{2/(\kappa x - 1)} a^{-1}$ and the PSDF along the volcanic front in Honshu island. Beneath the depth of 60 km, we cannot recognize any clear lateral variations because of the small number of data. Fig. 13 shows $B_{\text{freq}}$ distribution calculated from the inversion results. Even though the $B_{\text{freq}}$ values in our result tend to be smaller than the observed ones, our approach appropriately reconstructs the lateral variation of $B_{\text{freq}}$ and its focal depth dependence. This implies that the spatial distribution of $\kappa$ is reasonably estimated to describe the peak delay times in all frequency bands. Even if we change the weighting factors in the ranges of $200 \leq w_\kappa \leq 300$, $3.8 \leq w_\varepsilon \leq 5.0$ and $3.0 \leq w_{B_{\text{freq}}} \leq 5.0$, the overall characteristics of the inversion results are almost preserved as long as the objective function at the end of the second step is sufficiently small.

In the analyses using the GA, the standard deviation of the results can be evaluated by considering the covariance matrix using an \textit{a posteriori} Gaussian probability density function (e.g. Nakamichi \textit{et al.} 2003; Tarantola 2005). In this study, we evaluate the standard deviation using the genes having small squared residuals within 10 per cent of the best-fit model. We note that all of the genes satisfying this condition are found in the second step. The standard deviations (Fig. 14) do not show large lateral variations at all depth ranges, and the maximum standard deviations of the three parameters are 0.05 for $\kappa$, 0.06 for $\log(e^{2/(\kappa x - 1)} a^{-1})$, and 1.3 for $\log(\text{PSDF})$. The standard deviations for $\kappa$ and $\log(e^{2/(\kappa x - 1)} a^{-1})$ are small enough to discuss the inversion results quantitatively. The maximum standard deviation of $\log(\text{PSDF})$ seems to be large, but we cannot recognize any correlations between the large PSDF and the large uncertainties. This result may allow us to discuss the spatial distribution of the PSDF qualitatively.

All of the quantities of our results show a smooth distribution in the horizontal direction, but the variation in the vertical cross-sections is relatively large (Fig. 12). We conduct a few inversion analyses, incorporating a smoothing constraint in the vertical direction. Although the smoothness in the vertical direction is improved, $F(B_{\text{freq}}, \sigma_{B_{\text{freq}}})$ becomes larger and the observed frequency dependence disappears. This is not preferable for the peak delay time analyses, especially for the model assuming the von Kármán-type PSDF. It is probably better to use a smoothing constraint in the vertical direction, but the distributions of stations and hypocenters could not allow us to use smaller blocks in the vertical direction. Accordingly, this study uses the smoothing constraints only for the horizontal direction. In the following, we focus only on the lateral variation of random inhomogeneities in each depth layer.
5.2 Detailed characteristics of inversion result

The $\kappa$ values in the shallowest layer (0–20 km) are 0.28–0.68, with gradual change in the horizontal direction. The small values of $\kappa$ ($\kappa \leq 0.35$) are detected beneath the Quaternary volcanoes. The PSDF at the 0–20 km depth shows relatively large values $\sim 3.0 \times 10^{-6}$ km$^{-3}$, except in the backarc side of Honshu island. A large anomaly of $\varepsilon^{2/(p\kappa-1)}a^{-1}$ is found beneath thick sediments of Sendai plain, which may reflect the reverberation in the sediment layers, as pointed out by Petukhin et al. (2006).

The regions beneath the Quaternary volcanoes are characterized by small $\kappa$ and small $\varepsilon^{2/(p\kappa-1)}a^{-1}$ at the 20–60 km depth. The $\kappa$ value beneath the Quaternary volcanoes decreases to 0.20, and $\varepsilon^{2/(p\kappa-1)}a^{-1}$ is $2.0 \times 10^{-6} \sim 2.0 \times 10^{-4}$ km$^{-1}$. For example, the PSDF beneath Kurikoma volcano is $P(m) \sim 0.020 m^{-3.7}$ km$^3$ at the depth of 20–40 km, and that beneath the Iwate region at the 40–60 km depth is $P(m) \sim 0.058 m^{-3.7}$ km$^3$ for $0.5 < m < 50$ km$^{-1}$. The small values of $\kappa$ agree with the strong frequency dependence of the peak delay times for the waves propagating beneath the Quaternary volcanoes (Takahashi et al. 2007). In the forearc side, the random velocity inhomogeneities are estimated as $P(m) \sim 0.008 m^{-4.2}$ km$^3$. The difference in the PSDF in the forearc and backarc sides becomes small at small wavenumbers. This characteristic is consistent with the observed fact that the difference in the envelopes in the lower-frequency bands becomes unclear between the forearc side and the backarc side (e.g. Obara & Sato 1995; Takahashi et al. 2007). Saito

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Figure 11. Resultant spatial variation of $\kappa$, $\varepsilon^{2/(p\kappa-1)}a^{-1}$ and PSDF (at $m = 15$ km$^{-1}$, for $a = 5$ km) at three different depth ranges: (a) 0–20 km, (b) 20–40 km and (c) 40–60 km. Grey-coloured blocks correspond to those having a few ray paths less than 5 and those in which the number of ray paths having long travel distance ($\geq 80$ km) from the source to the block is less than 5. Open triangles are the Quaternary volcanoes.
et al. (2005) estimated the PSDF in the forearc side as $0.004 m^{-5.0} \sim 0.01 m^{-4.0} km^3$ by assuming the spatially uniform distribution of the von Kármán-type PSDF. Our result in the forearc side agrees with their result.

The western Hidaka area shows large $\varepsilon^{2/(\rho \kappa - 1)}a^{-1}$, large $\kappa$ and large PSDF at the depth of 20–40 km. The $\varepsilon^{2/(\rho \kappa - 1)}a^{-1}$ is up to $10^{-3} km^{-1}$, and the $\kappa$ is approximately 0.6. The PSDF in this region is $P(m) \sim 0.067 m^{-4.2} km^3$. The envelopes of seismic waves propagating through this region are strongly broadened, regardless of the frequency bands (Takahashi et al. 2007). This observation is consistent with the large $\kappa$ and large $\varepsilon^{2/(\rho \kappa - 1)}a^{-1}$ values. The eastern off Aomori region indicates small $\kappa$ and moderate $\varepsilon^{2/(\rho \kappa - 1)}a^{-1}$ values at the depth of 40–60 km. The east off Aomori region indicates $\kappa = 0.45 \sim 0.55$ and $\varepsilon^{2/(\rho \kappa - 1)}a^{-1} \sim 2.0 \times 10^{-3} km^{-1}$. The PSDF at short wavelengths becomes $P(m) \sim 0.019 m^{-3.7} km^3$. Even though there is no Quaternary volcano, the PSDF characterized by a weak spectral decay is similar to that beneath the Quaternary volcanoes.

We can also find some large PSDF regions along the eastern edge of the study area, where the peak delay times are usually less than a few seconds because of short travel distances. If we consider the rms residual of the peak delay time, it may be difficult to conclude that these anomalies are meaningful. In addition, if we take account of the accuracy of focal depths and the scatter of the peak delay times due to amplitude fluctuation of envelopes,
we should restrain ourselves from discussing the details of these strongly inhomogeneous regions along the eastern edge of the study area.

6 DISCUSSION

6.1 Comparison with other geophysical observations and possible interpretations

In northeastern Japan, it has been recognized that the forearc side of the volcanic front is characterized by weak inhomogeneities and weak attenuation and that the backarc side has strong inhomogeneities and strong attenuation (e.g. Umino & Hasegawa 1984; Obara & Sato 1995; Yoshimoto et al. 2006). Our inversion analysis reveals the clear spatial variations of random inhomogeneities in both the forearc and the backarc sides and suggests that the spectral decay is an important characteristic of the random inhomogeneities. In the following, we examine the relation between the spatial distributions of random inhomogeneities and other geophysical observations then infer possible interpretations for the origins of random inhomogeneities.

The strong inhomogeneities beneath the Quaternary volcanoes are approximately located in the low-velocity and high \( V_p/V_s \) regions detected by the traveltome tomography (e.g. Zhao et al. 1992; Nakajima et al. 2001a). Low-frequency earthquakes have been also observed near these low-velocity regions (e.g. Nakajima et al. 2001b). As a possible origin of the low-velocity anomalies and low-frequency earthquakes, the existence of the magma–diapir and/or aqueous fluid has been inferred from the studies on the subduction zone dynamics and genesis of the Quaternary volcanoes (e.g. Tamura et al. 2002; Hasegawa & Nakajima 2004; Nakajima et al. 2005). We speculate that such liquid inclusions are the origin of the strong velocity fluctuation beneath the Quaternary volcanoes. Similar interpretation of the relation to liquid inclusions has been already proposed by Obara & Sato (1995). Our inversion result makes clearer the relation between the low-velocity regions and the strongly inhomogeneous regions. Attenuation tomography in northeastern Japan (e.g. Nakamura & Uetake 2002) imaged high-attenuation regions beneath the volcanic front. Our result suggests that such a strong attenuation cannot be assigned only for the intrinsic absorption. Petukhin & Kagawa (2007) imaged a high-attenuation region in a high \( V_p/V_s \) region in a seismic wave reflector in the southwestern Japan. Their result also suggests that the
random velocity fluctuation is an important property of a medium that possibly relates the velocity and attenuation structures. The western Hidaka area shows high seismicity at the depth of 20–40 km (e.g. Katsumata et al. 2003). We infer that fractured structures due to seismicity generate the strong inhomogeneities in this region. The east off Aomori region is located near the rupture area of the 1968 Tokachi-oki earthquake (M 8.2). In addition, this area almost corresponds to weak anomaly of low $V_s$ and high Poisson ratio region according to the traveltime tomography (Zhao et al. 2007). These strong inhomogeneities are characterized by the

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**Figure 14.** Map view of the standard deviations of the estimated parameters: (a) $\kappa$, (b) $\log(\frac{\varepsilon}{\varepsilon_0})$ and (c) log(PSDF) (at $m = 15$ km$^{-1}$ for $a = 5$ km) at three depth ranges: 0–20 km, 20–40 km and 40–60 km. Open triangles are the Quaternary volcanoes.
small $\kappa$, and the relation between the strong inhomogeneities and the velocity structure seems to be similar to that in the regions beneath the Quaternary volcanoes. This relation may be important to interpret the origin of random inhomogeneities. However, if we consider the ray path distribution, smoothing constraints and uncertainty of focal depths of offshore earthquakes, we may need to infer a possibility of the effect of fractured structure generated by large earthquakes. The strong inhomogeneities associated to large earthquakes have been reported from the coda wave analyses (Nishigami 1991; Asano & Hasegawa 2004). For example, Asano & Hasegawa (2004) analysed coda wave envelopes (4.5–9.0 Hz) to estimate the spatial distribution of scattering coefficients around the fault zone of a large earthquake ($M$ 7.3). They found large scattering coefficients along the fault of the main shock in the upper crust and inferred the existence of a damaged zone along the fault. Since the result in the western Hidaka region does not necessarily mean that all of the fractured structures are characterized by a large $\kappa$, we should not reject the possibility of its relation to large earthquakes in the east off Aomori region.

6.2 Possible developments of inversion analysis

Our results clearly show that the spatial distribution of random velocity inhomogeneities is strongly related to the seismotectonic conditions in northeastern Japan. We reveal that the statistical parameters of random inhomogeneities are different among the regions that commonly show strong inhomogeneities in short wavelengths ($m > 15$ km$^{-1}$). These results imply that the statistical parameters characterizing the von Kármán-type PSDFs are useful to discuss the mutual relations of the earth medium inhomogeneity and seismotectonic conditions. However, we should note that there are some possible improvements on our method; for example, we need to resolve the systematic underestimation of $B_{\text{eq}}$ in the backarc side. Such improvements will be necessary to discuss the details of random inhomogeneities in the earth.

The Markov approximation assumes weakly inhomogeneous media characterized by specific types of PSDFs and can take account of anisotropy only for limited situations (e.g. Saito 2006; Saito 2008). The von Kármán-type PSDFs can be the origin of frequency dependence and provide a quantitative relation between the peak delay time and the parameters of the PSDF. However, the random inhomogeneities in the earth will not be always weak and will not be described only by isotropic von Kármán-type PSDFs. For example, Margerin & Nolet (2003) proposed to characterize the PSDF of random inhomogeneities in the lower mantle as $P(m) \propto m^{-a}$ from the PKP precursor analysis at 0.4–2.5 Hz. Their PSDF corresponds to the case of $\kappa = 0.0$, that is, a weak spectral gradient compared with that beneath the Quaternary volcanoes. If we extrapolate their PSDFs at large wavenumbers, we can expect significantly strong inhomogeneities at large wavenumbers. This result implies that large-angle scattering is not negligible. The relative ratio of forward- to wide-angle scattering depends on the wavelength of incident wave and the correlation distance of random inhomogeneities, according to the Born approximation (e.g. Sato & Fehler 1998, p. 106). We may say that the comparison of the Markov approximation and the radiative transfer approach is necessary to improve the modelling of random inhomogeneities in the lithosphere. We also note that incoherent scattering effects in a tube around the unperturbed ray will be necessary, especially beneath strongly inhomogeneous regions. This study evaluates the peak delay time using the random inhomogeneities along the unperturbed ray path, but we can expect that the diameter of this tube is larger than that of the sensitivity kernel for coherent waves in seismic velocity tomography (e.g. Dahlen et al. 2000).

For some additional constraints on the lithospheric inhomogeneities, we also need the analyses of direct $P$-wave envelopes to measure the lithospheric inhomogeneity. In regional distances, it is difficult to apply our peak delay time analysis for $P$ waves because the strong $P$ coda usually masks the maximum amplitude arrival of direct $P$ wave. However, at teleseismic distances, Kubanza et al. (2006, 2007) evaluated the lithospheric inhomogeneities and discussed a relation with seismotectonic conditions from the partition of $P$-wave energy into the transverse component on the basis of the Markov approximation for vector waves. A comparison of envelope analyses of $P$ and $S$ waves may help us to interpret the random inhomogeneities in the lithosphere. It will be also necessary to introduce $P$–$S$ conversions in the envelope synthesis of three-component vector waves (e.g. Przybilla & Korn 2008).

This study only analyses the peak delay times and infers a relation of random inhomogeneities to the liquid inclusions beneath the Quaternary volcanoes. Since such liquid inclusions will strongly absorb the wave energy, we will need to reveal the spatial distribution of the intrinsic absorption too. There is an analysis of the whole $S$-wave envelopes including coda of small earthquakes: the multiple lapse time window analysis simultaneously evaluates the scattering loss and intrinsic absorption in the earth medium (Fehler et al. 1992; Hoshina 1993). Recently deployed high-density seismic networks will make it possible to map the spatial variation of both the scattering loss and the intrinsic absorption. It will be helpful to use both the envelope analysis of direct waves developed here and the whole $S$-wave envelope analysis for understanding the precise structures of the crust and uppermost mantle.

7 Conclusions

This study proposes a new practical approach for an inversion analysis of the peak delay times of $S$ wave of local microearthquakes at high frequencies to estimate the spatial distribution of random velocity inhomogeneities by assuming 3-D isotropic von Kármán-type PSDFs. Two unknown parameters $\kappa$ and $b_{\text{PSDF}}^{\text{obs}}$ are used to quantify the PSDF at wavelengths shorter than the correlation distance. The parameter $\kappa$ is related to the spectral decay, and the parameter $b_{\text{PSDF}}^{\text{obs}}$ can be regarded as a constraint on spectral amplitudes at wavelengths shorter than the correlation distance. For a stable estimation of the two parameters, we propose a two-step approach that includes an explicit constraint on the frequency dependence of the peak delay times. The frequency dependence is evaluated from the regression analysis using the peak delay times observed at each station. The first step constructs an initial model...
that reflects the spatial changes of the frequency dependence and the peak delay times in the reference frequency band (4–8 Hz). The second step minimizes the residual of the peak delay times at all frequency bands (2–4 Hz, 4–8 Hz, 8–16 Hz and 16–32 Hz). The synthetic tests with and without the constraint for the frequency dependence make it clear that our two-step approach works effectively to improve the estimation of the two parameters.

We apply the proposed two-step inversion method to the peak delay times in northeastern Japan and reveal a clear regional variation of random velocity inhomogeneities related to the Quaternary volcano distribution and seismicity. Beneath the Quaternary volcanoes, the PSDFs of random inhomogeneities $P(m)$ are estimated as $0.017m^{-3.3} \text{km}^{-1} \sim 0.035m^{-4.0} \text{km}^{-1}$ at $0.5 < m < 50 \text{ km}^{-1}$ for the case in which we assume $a = 5$ km. A remarkable characteristic of these regions is the significantly weak spectral decay (small $\alpha$). These strongly inhomogeneous regions are almost located in the low-velocity and high-$V_p/V_s$ regions detected by the traveltime tomography. This coincidence implies that the possible origins of strong inhomogeneities are the existence of magma-diaper and/or aqueous fluid. The western Hidaka region in Hokkaido showing high seismicity is characterized by a steep gradient of spectral decay: $P(m) = 0.023m^{-4.2} \sim 0.050m^{-4.2} \text{km}^{-1}$ for $0.5 < m < 50 \text{ km}^{-1}$. We speculate a fractured structure due to seismicity as a possible origin of strong inhomogeneities. Our approach detects the difference in the statistical parameters among these strongly inhomogeneous regions that commonly indicate large PSDF at large wavenumbers ($m > 15 \text{ km}^{-1}$). Our result implies that the statistical parameters of random inhomogeneities can be used for the discussion on the seismotectonic conditions. Even though our approach relies on the Markov approximation by assuming an isotropic von Kármán-type PSDF, our current approach and result can be a basis to develop some approaches estimating the lithospheric structures.

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APPENDIX: DETAILS OF INVERSION PROCEDURE

We summarize the implementation of the neighbourhood algorithm (NA) into the genetic algorithm (GA) and modifications of crossover and mutation processes.

The NA is implemented at the end of each generation of the GA by the following procedure. We pick up $N_{NA}$ genes having better fitness, and the average and standard deviation for the two parameters in each block are determined from these $N_{NA}$ genes. Then, the two parameters of new $N_{NA}$ genes are randomly generated within the range of the standard deviation for each block. The $N_{NA}$ genes having worse fitness are replaced by these newly generated genes. If the standard deviation becomes zero as the iteration proceeds, the parameters in new $N_{NA}$ genes are randomly generated within ±5 per cent of the value of the selected $N_{NA}$ genes. The value of ±5 per cent is chosen after some trials to avoid both the local minimum and the insufficient convergence of inversion. In this study, the 10 genes among 100 genes are used in the NA process. The NA process runs in parallel with this reproduction as follows: new 90 genes are reproduced from the original 100 genes by reproduction, and the 10 genes are generated by applying the NA for original genes. Even though we do not use the Voronoi cell to define the neighbourhood of the best-fit model (e.g. Sambridge 1999) because of the large number of unknown parameters, we confirmed that this operation significantly improves the convergence of inversion.

Modifications for crossover and mutation are introduced to achieve sufficient convergences of all terms in objective functions with small weighting factors for the Laplacian constraints. The aim of the modifications is that these processes produce and save the smooth distribution of the parameters at high probability without any a priori information or any human-induced operation, since the crossover and mutation processes tend to disturb the smooth distribution of the parameters, regardless of the Laplacian constraints. In the mutation process, the probability of a mutation decreases as the generation increases to suppress the disturbance of smoothness. In this study, the mutation happens for the reproduced genes under the mutation process, the probability of a mutation decreases as the generation increases to suppress the disturbance of smoothness. In this study, the mutation happens for the reproduced genes under the

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every 500 steps. The crossover is modified by introducing a parameter for each block to quantify the difference in the parameter with its neighbours as

\[ s = \sum_{i,j=\pm 1} \{ w_\kappa (\nabla^2 \kappa_{i,j})^2 + w_\epsilon (\nabla^2 \log(\varepsilon^{2/(p(x)-1)} a^{-1}))_{i,j} \}^2. \]

The subscripts \( i \) and \( j \) represent the indices of block location in a horizontal plane measured from each block. The usual crossover randomly exchanges the parameters between two genes. This modified crossover is conducted under the following rule under a probability \( p_{c,x} \): one of the new genes is composed with the blocks having smaller values of \( s \), and another one is a group of larger values of \( s \). We can expect that the former one has a larger possibility to be a smooth distribution compared with that generated by the usual crossover, although this operation can also generate a rougher distribution because the values of the neighbour blocks can be changed. If this process generates excessively smooth structures that cannot fit the observed peak delay times, the reproduction in the GA excludes such unreasonable genes. We set \( p_{c,x} = 0.6 \) after examining the convergence rate and the risk that the random search is trapped in local minima. Comparing the inversion results with and without this process, we confirm that this modified crossover algorithm effectively works to generate a smooth distribution of the parameters with high probability and achieve stable convergences of \( t_p \), \( B_{\text{seq}} \) and the Laplacian constraints.